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## English summary

The dynamics of human crowds is an interesting, but highly complex topic, and phenomena have been observed ranging from well-organized structure formation leading to quasi steady-state equilibrium to violent and turbulent flows resulting in catastrophic scenarios with large numbers of casualties. This report reviews fifty years of scientific work on crowd dynamics. Although the aim of this report is to gain insight into the modelling of exceptional crowd events such as demonstrations and riots, a large portion of the described research deals with normal state, pedestrian traffic. This is partly due to the fact that the open literature on crowd modelling is dominated by this particular class of applications, but also because treating normal behaviour crowd dynamics is a necessary first step in modelling also the more extreme cases of crowd dynamics. The main part of the report is devoted to classifying and describing different theoretical crowd models. However, a separate section on relevant experimental work is also included. This report marks the first step towards developing a new crowd simulation tool applicable to scenarios involving human crowd and less-lethal weapons interactions.

## Sammendrag

Dynamikken til ei menneskemengd er eit interessant, men svært komplekst tema. Observerte fenomen knytt til menneskemengder varierar frå danning av velorganiserte strukturar som fører til kvasistasjonære jamvektstilstandar, til valdelege og turbulente straumar som resulterer i katastrofale hendingar med mange omkomne. Denne rapporten presenterar ein gjennomgang av vitskapleg arbeid gjennomført dei siste femti åra på temaet dynamikk i menneskemengder. Hovudmålet med rapporten er å få innsikt i modellering av eksepsjonelle hendingar der menneskemengder er involvert, slik som demonstrasjonar og opptøyar. Likevel fokuserer store delar av forskinga som vert skildra det vi kan kalle fotgjengartrafikk eller liknande former for normale fenomen i ei menneskemengd. Dette skuldast dels det faktum at svært mykje av den opne litteraturen på temaet dynamikk i menneskemengder blir dominert av akkurat denne type problem. Men i tillegg er studia av dynamikk i menneskemengder i normaltilstand viktig som eit fyrste steg med tanke på å forstå dynamikken til menneskemengder i meir ekstreme situasjonar òg. Hovuddelane av rapporten tek for seg klassifisering og skildring av teoretiske modellar av menneskemengder. I tillegg, skildrar ein eigen del relevant eksperimentelt arbeid på området. Denne rapporten markerer fyrste steg mot å utvikle eit nytt verkty for simulering av menneskemengder som kan nyttast på situasjonar som involverer vekselverknad mellom ei menneskemengd og mindre-dødelege våpen.

# Contents

# 1 Introduction

The dynamics of human crowds is an interesting, but highly complex topic, and phenomena have been observed ranging from well-organized structure formation leading to quasi steady-state equilibrium to violent and turbulent flows resulting in catastrophic scenarios with large numbers of casualties. Optimization of pedestrian flow is linked to the desire for increased efficiency and has been studied empirically for nearly fifty years (Fruin, 1971; Hankin & Wright, 1958; Older, 1968; Weidmann, 1992). Numerous crowd disaster events are known from history, and the table in Fig. 1.2 shows a selection of disasters occurring over a period of 35 years (Helbing & Johansson, 2009). All events in the list were fatal with the number of deaths ranging from tens to over a thousand people. Still none of the events were originally related to fires, bomb attacks, train or plane accidents, or riots. The Wai tragedy in 2005 increased in size after angry pilgrims, in response to relatives and friends already being trampled to death, set shops on fire. The Ghana stadium disaster in 2001 can largely be attributed to the fact that the police used plastic bullets and tear gas in response to disappointed football fans throwing plastic seats and bottles onto the pitch. The 2006 event in Yemen occurred at a political rally, not a violent riot. According to a news bulletin in the Guardian, most of the dead were believed to be schoolchildren and teenagers.

*Figure 1.1 Picture taken at the Love Parade crowd disaster in Duisburg, Germany in 2010. In total 21 people died and more than 500 people were injured.*

If we were to include crowd disasters which started with a fire, a bomb attack, accidents, or a riot, the list of human tragedies where crowd dynamics plays a role becomes substantially longer. It quickly becomes clear that crowd dynamics deals with many aspects of human nature. But in addition, it is also quite clear that crowd dynamics is related to phenomena, such as fluid flow and many-particle physics, studied within the framework of mathematics and the natural sciences.

| Date | Place | Venue | Deaths | Reason |
|---|---|---|---|---|
| 1971 | Ibrox, UK | Stadium | 66 | Collapse of barriers |
| 1974 | Cairo, Egypt | Stadium | 48 | Crowds break barriers |
| 1982 | Moscow, USSR | Stadium | 340 | Re-entering fans after last minute goal |
| 1988 | Katmandu, Nepal | Stadium | 93 | Stampede due to hailstorm |
| 1989 | Hillsborough, Sheffield, UK | Stadium | 96 | Fans trying to force their way into the stadium |
| 1990 | New York City | Bronx | 87 | Illegal happy land social club |
| 1990 | Mena, Saudi Arabia | Pedestrian Tunnel | 1426 | Overcrowding |
| 1994 | Mena, Saudi Arabia | Jamarat Bridge | 266 | Overcrowding |
| 1996 | Guatemala City, Guatemala | Stadium | 83 | Fans trying to force their way into the stadium |
| 1998 | Mena, Saudi Arabia | | 118 | Overcrowding |
| 1999 | Kerala, India | Hindu Shrine | 51 | Collapse of parts of the shrine |
| 1999 | Minsk, Belarus | Subway Station | 53 | Heavy rain at rock concert |
| 2001 | Ghana, West Africa | Stadium | > 100 | Panic triggered by tear gas |
| 2004 | Mena, Saudi Arabia | Jamarat Bridge | 251 | Overcrowding |
| 2005 | Wai, India | Religious Procession | 150 | Overcrowding (and fire) |
| 2005 | Bagdad, Iraque | Religious Procession | > 640 | Rumors regarding suicide bomber |
| 2005 | Chennai, India | Disaster Area | 42 | Rush for flood relief supplies |
| 2006 | Mena, Saudi Arabia | Jamarat Bridge | 363 | Overcrowding |
| 2006 | Pilippines | Stadium | 79 | Rush for game show tickets |
| 2006 | Ibb, Yemen | Stadium | 51 | Rally for Yemeni president |

*Figure 1.2 Selection of some major crowd disasters in the period 1971-2006. The table is taken from Helbing & Johansson (2009).*

## 1.1 Collective behaviour

Let us assume we have a system consisting of many similar entities, and that the interactions between the entities under certain conditions can lead to transitions in the state of the system. If a transition causes the entities to adopt a pattern of behaviour almost completely determined by the collective effects due to the other entities in the system, then we have an example of collective behaviour. An important aspect of collective behaviour is that the dynamics of a single entity is dominated by the influence of the other entities and that behaviour therefore can be radically different from what one would observe when the entity is left on its own. Fig. 1.3 shows a handful of examples of collective motion involving living, "self-propelled" entities (Vicsek & Zafeiris, 2012). But even non-living entities, such as molecules or self-propelled particles can exhibit collective behaviour (Czirók, Stanley & Vicsek, 1997).

Human crowds of sufficiently high density exhibit features of collective behaviour, such as arching near bottlenecks (Helbing, Farkas & Vicsek, 2000), stop-and-go waves evolving into turbulence (Helbing, Johansson & Al-Abideen, 2007), lane formation (Helbing et al., 2001), and freezing

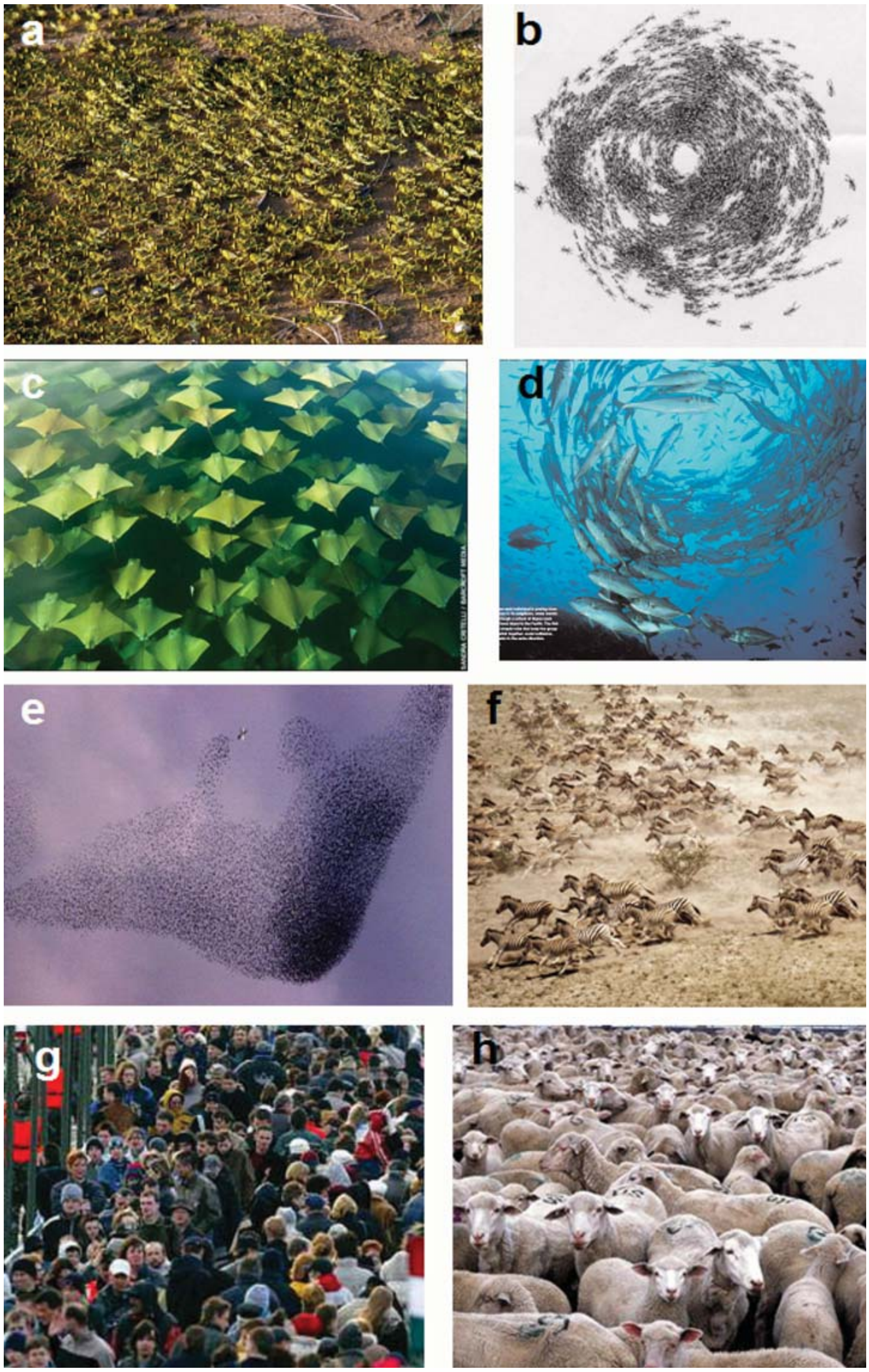


*Figure 1.3* *Examples of collective behaviour among animals and humans:* **(a)** *Wingless locusts marching.* **(b)** *A rotating colony of ants.* **(c)** *A three-dimensional array of golden rays.* **(d)** *A vortical structure of fish.* **(e)** *Starlings in flight facing a predator bird.* **(f)** *A herd of zebras.* **(g)** *People spontaneously ordered into channels of uni-directional flow.* **(h)** *A herd of sheep in motion. The figure is taken from Vicsek & Zafeiris (2012).*

by heating (Helbing, Farkas & Vicsek, 1999). These are quite generic phenomena of collective behaviour which a human crowd can have in common with a set of non-living particles. Trail formation is another example of collective behavior observed in relation to movement patterns among both animals and humans (Helbing et al., 2001): A trail does not simply follow a straight line, and the trail has not been designed by individual animals or humans. Instead, a trail is the result of some kind of underlying optimization process in which a number of individuals over a relatively long period of time have contributed to the result. In all of these examples, the behaviour is not primarily the result of an intelligent thought process, but rather the result of a complex coupling of the collective state and the reaction this causes at the level of the individual. Although this gives a certain foundation for the idea of modeling human crowd behaviour using a mathematical approach, a question still remains: What happens when the density in the crowd drops?

## 1.2 Modelling of socio-economic systems

Gradually, as the density in a crowd drops, the influence of other people on the motion of indi-vidiuals decreases. The ideas, desires, and motivations of each person in the crowd start to have a stronger and stronger impact on the dynamics. At this point, we have a socio-economic system which is far more complicated to model than a densely populated crowd: (i) The number of factors playing a significant role in the decision making process of individuals is in practice endless. (ii) Randomness often plays an important role. (iii) Typically, there is no ensemble of equivalent systems since no two groups of people are truly identical. (iv) Non-linear models with many variables can be expected to exhibit phenomena like hysteresis and chaos and may therefore be highly dependent on the exact specification of parameters. (v) And finally, with many variables comes the inherent problem of separating the effect of different variables from each other. The modelling of socio-economic systems has been discussed extensively by Helbing (2010). He divides relevant modelling approaches into 3 categories: Qualitative descriptions, detailed models, and simple models.

### 1.2.1 Qualitative descriptions

Due to the challenges associated with socio-economic modelling, it is quite common among social scientists to reject the modelling approach altogether. This is justified by the view that all models over-simplify the socio-economic processes. This leaves mainly one possible research approach: To work out narratives that describe real-life events as detailed as possible, compare descriptions of several similar events, and finally draw conclusions regarding the systems on the basis of comparison. This approach is clearly fruitful in providing an overview of the problem and in giving the researchers a qualitative understanding of dynamics of the system. However, it can be difficult to extract general guidelines based on qualitative descriptions. And it will typically be even harder to gain in-depth knowledge on threshold values of important quantities such as density and velocities.

### 1.2.2 Detailed models

The next approach, also quite commonly used in psychological and social research, is to develop highly detailed models which attempt to include as many aspects as possible of the system under consideration. There is an element of simplification in that one includes only features that are assumed to be characteristic to the relevant type of system. However, the number of variables and parameters is typically very large, and the parameters are often chosen to fit specific observations. As a consequence, such models might very well give realistic results in a certain range of applications. The problem is, however, that the input parameters have been so finely tuned to the applications being studied that no really new knowledge is gained. With a sufficiently large number of tuned parameters, a model can be structured that fits any phenomena. Also, if the model is comparable to the real-life system in complexity, then interpreting the results of the model will be just as difficult as interpreting real-life events.

### 1.2.3 Simple models

In simple models, one tries to reduce the number of variables and parameters down to a minimum. The hope is that dominating processes in socio-economic systems can still be recreated. The results from applying such a model can more easily be interpreted and does not have to rely so heavily on input parameters that are difficult to calibrate in the first place. Features not explicitly described by the models can still in some sense be taken into account by applying a statistical approach. Critics still claim that a simplified model is likely to neglect features that are essential to the description of the system. It is also difficult to keep the model simple as one naturally wants better fit to observations.

# 2 Classification and evaluation of models

Crowd modelling has in recent years become an important tool in studying the dynamics of human crowds. It has become a key design issue in applications ranging from military simulation, safety engineering, architectural design, and digital entertainment. Looking beyond the specific applications, Zhou et al. (2010) sort crowd models primarily based on crowd size and relevant time scales. Small-and medium-sized crowds include from a few tens up to roughly a thousand people, while a large crowd model can include tens of thousand people or more. Time scales can also vary by several orders of magnitude. Short time scale phenomena often deal with the movement patterns changing considerably within seconds or minutes. Long time scale phenomena can include social or psychological changes taking years to develop. Fig. 2.1 shows how different modeling approaches and different application categories is distributed in the two-dimensional parameter space.

**Fluid dynamical models:** Based on the assumption that the dynamics of individuals become unimportant to the description of the dynamics of a large, densely populated crowd, crowd models based on Navier-Stokes equations have been developed. These models usually neglect the description of individuals altogether (Bradley, 1993). One problem with these models is that the

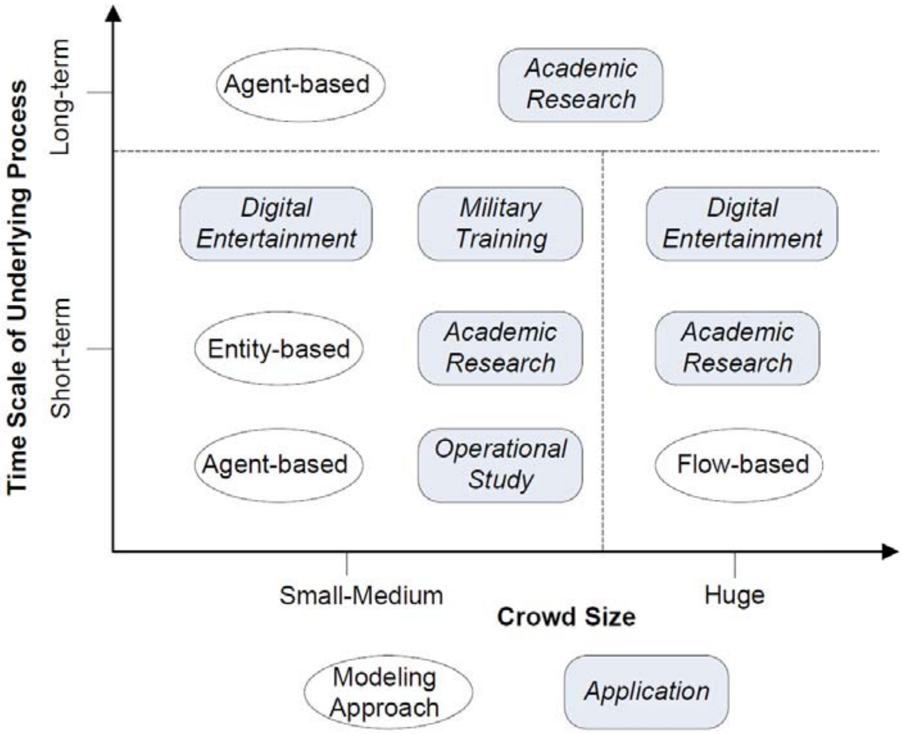


*Figure 2.1 Classification of crowd models based on crowd size and time scale (taken from Zhou et al., 2010).*

basic assumption limits the applicability of the models quite severely. Another issue is the fact that Navier-Stokes type equations are non-linear and coupled and thus fairly difficult to solve both efficiently and accurately.

**Particle models:** Particle models, also referred to as entity-based models, represent the simpler option among the discrete model types. People are treated more or less as identical particles with little or no internal structure. The dynamics of the particles is either determined by forces that reflect various physical, social, and psychological influences, or through ruled-based algorithms that consider the local distribution of particles. The former is known as *social force models* (Helbing & Molnár, 1995), while the latter approach is most commonly applied in relation to so-called *cellular automata models* (Burstedde et al., 2001).

**Agent-based models:** When we start referring to the model entities as agents rather than particles, this indicates that the complexity in the modelling of individuals has been increased. Although there is not a clearly defined distinction between the two, agent-based models incorporate models of internal processes, both physical and psychological, which is not included in particle models. This usually means that the internal state of the agents vary in time according to social interactions with other agents. Social relation between agents can also be of importance. Agent-based models can be seen as an extension to the social force approach (Braun, Bodmann & Musse, 2005), rely on rule-based action (Wijermans, 2011), or be some kind of combination of the two (Korhonen et al., 2008a).

Zhou et al. (2010) discusses different criteria with which one can evaluate the different crowd models: **(a)** Flexibility refers to a model's ability to adapt to different situations. **(b)** Extensibility indicates how easily the model can accommodate new features. **(c)** Efficiency concerns the Cpu-time needed to execute a crowd simulation for a given scenario. **(d)** Scalability tells us how much execution time and memory usage increase with increasing crowd size. Strangely enough, Zhou et al. (2010) do not include believability as one of their criteria. They argue that believability is subjective and therefore not properly defined. However, it is reasonable to extend the list with the following two evaluation criteria: **(e)** Accuracy is a measure of how well a given model reproduces experimental data. **(f)** Robustness refers to how sensitive the simulation results are to uncertainties in the model parameters and how this compares to uncertainties in the experimental data.

# 3 Specific models

After having given this general overview over different approaches to crowd modelling and different types of modelling techniques, we will take a closer look at a 3 specific models which highlights important differences in numerical approaches. We will also briefly review a handful of other models which represent alternatives to the first three models. Finally in this section, we list some of the commercial crowd modelling software packages available. This review will limit itself to models relevant for simulating small- and medium-sized crowds over relatively short-term time scales.

## 3.1 The Helbing social force model

Perhaps the single most important contributor to the field of numerical crowd modelling, is Prof. Dirk Helding at ETH Zürich (formerly at Dresden University of Technology) and his collaborators. The concept of using a social force model was first presented in 1995 Helbing & Molnár (1995), and the models have been gradually revised and extended since then studying phenomena such as pedestrian transportation, panic in crowds, and crowd disasters (Helbing, Farkas & Vicsek, 2000; Helbing & Johansson, 2009; Helbing, Johansson & Al-Abideen, 2007; Helbing et al., 2001). Fig. 3.1 illustrates some of the results obtained with the Helbing model. Despite the unquestionable impact that the model has had on crowd modelling, the Helbing model has been criticized for over-simplifying the problem (Lakoba, Kaup & Finkelstein, 2005). The model we will describe in some detail here, is taken from Helbing et al. (2002).

The idea behind the Helbing force model is to model crowd behaviour by formulating forces which describe interaction between pairs of persons, interaction between a person and a solid wall, and the self-propelling force which represents a person's own will to move in a certain way. Each person, or agent if you will, $a$ has attributes such as mass ($m_a$), physical diameter ($d_a$), position ($\boldsymbol{r}_a$), and velocity ($\boldsymbol{v}_a$). The momentum equation for the agent can be written as

$$m_a \frac{d\boldsymbol{v}_a}{dt} = \sum_{b \in \mathcal{A}} \boldsymbol{f}_{ab}^{\text{soc}} + \sum_{b \in \mathcal{A}} \boldsymbol{f}_{ab}^{\text{con}} + \sum_{c \in \mathcal{B}} \boldsymbol{f}_{ac}^{\text{bnd}} + \boldsymbol{f}_a^{\text{will}}. \tag{3.1}$$

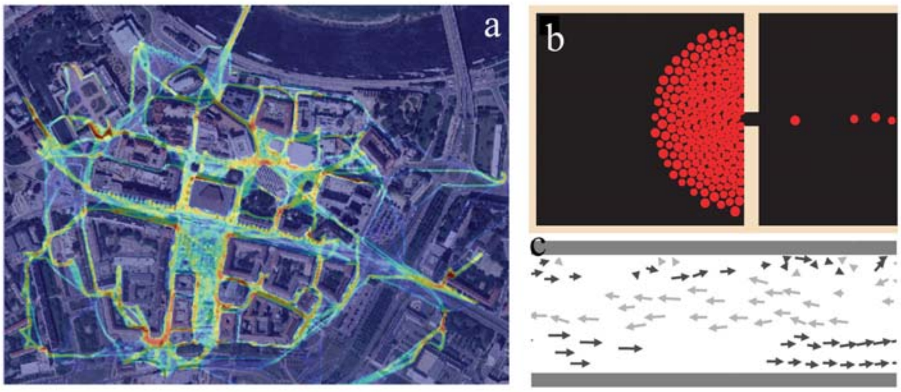


*Figure 3.1* *Crowd modeling results obtained with the Helbing model:* **(a)** *Density of pedestrians in a simulation of 30.000 agents in the city centre of Dresden, Germany (Johansson, Helbing & Shukla).* **(b)** *Simulation of pedestrians moving towards a 1-m wide exit of a room of size 15 m $\times$ 15 m (Helbing, Farkas & Vicsek, 2000).* **(c)** *Lane formation in crowds of oppositely moving agents (Helbing et al., 2001).*

The forces acting on agent $a$ are divided into 4 based on the type of interaction: $\boldsymbol{f}_{ab}^{\text{soc}}$ and $\boldsymbol{f}_{ab}^{\text{con}}$ represent social and contact force interactions, respectively, with neighbouring agent $b$ (where $\mathcal{A}$ is the set of all agents), $\boldsymbol{f}_{ac}^{\text{bnd}}$ denotes interactions with solid boundary element $c$ (where $\mathcal{B}$ is the set of all boundary elements), and $\boldsymbol{f}_{a}^{\text{will}}$ indicates the self-propelling will force. To simplify the discussion on inter-agent force, we also define the following quantities for a pair of agents $a$ and $b$:

$$\boldsymbol{r}_{ab} = \boldsymbol{r}_b - \boldsymbol{r}_a, \tag{3.2}$$

$$\hat{\boldsymbol{r}}_{ab} = \frac{\boldsymbol{r}_{ab}}{\|\boldsymbol{r}_{ab}\|}, \tag{3.3}$$

$$\hat{\boldsymbol{t}}_{ab} = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{r}}_{ab}, \tag{3.4}$$

$$\boldsymbol{v}_{ab} = \boldsymbol{v}_b - \boldsymbol{v}_a, \tag{3.5}$$

and

$$\hat{\boldsymbol{v}}_{ab} = \frac{\boldsymbol{v}_{ab}}{\|\boldsymbol{v}_{ab}\|}, \tag{3.6}$$

where $\hat{\boldsymbol{z}} \equiv [0, 0, 1]$ is the constant unit vector out of the computational plane.

### 3.1.1 Social inter-agent forces

The social inter-agent force describes primarily the tendency of people to keep a certain minimum distance to other people, known as the "territorial effect". If agents are part of a group, e.g. a family or group of friends, this is proposed modelled using an attraction force which will counteract any splitting of the group. The social force between agents $a$ and $b$ is formulated as

$$\boldsymbol{f}_{ab}^{\text{soc}} = -A_a \exp\left(\frac{d_{ab} - r_{ab}}{B_a}\right) \left[\lambda_a + (1 - \lambda_a)\frac{1 + \cos\phi_{ab}}{2}\right] \hat{\boldsymbol{r}}_{ab}, \tag{3.7}$$

where $A_a$ is the interaction strength at the limit of physical contact and $B_a$ is the interaction range. In the Helbing model these parameters are constants typical equal to 2000 N and 0.08 m, respectively. The parameter $\lambda_a$ controls the anisotropy since $\cos\phi_{ab} = \hat{\boldsymbol{r}}_{ab} \cdot \boldsymbol{v}_a / \|\boldsymbol{v}_a\|$, and $d_{ab}$ is the mean physical diameter of the two agents, typically in the range 0.5-0.7 m. Note that the interaction range is often chosen to be much smaller than the physical diameter. This implies that the social force has a short range, the number of interactions per agent is small or moderate (in the case of very high densities), and the modelling of applications where relatively large velocity differences can occur might not be realistic (Lakoba, Kaup & Finkelstein, 2005).

### 3.1.2 Contact inter-agent forces

When the distance between two agents becomes equal to the corresponding mean physical diameter, the two agents come in direct contact. Normally, this should only occur at high crowd densities or in situations of panic. In this case, the Helbing-model assumes a body force counteracting body compression and a frictional force hindering tangential motion. The form of the physical force is

$$\boldsymbol{f}_{ab}^{\text{con}} = [\kappa_{\|} \hat{\boldsymbol{r}}_{ab} + \kappa_{\perp} (\boldsymbol{v}_{ab} \cdot \hat{\boldsymbol{t}}_{ab}) \hat{\boldsymbol{t}}_{ab}] \Theta(r_{ab} - d_{ab}), \tag{3.8}$$

where $\kappa_{\|} = 1.2 \cdot 10^5 \text{kgs}^{-2}$, $\kappa_{\perp} = 2.4 \cdot 10^5 \text{kgm}^{-1}\text{s}^{-1}$, and $\Theta(z) = zH(z)$ (where $H(z)$ is the Heaviside function which is 0 for $z < 0$ and 1 otherwise).

### 3.1.3 Will force

The self-propelling force, or will force, in the Helbing model uses a very simple formulation in that it only depends on the difference between the desired velocity $\boldsymbol{u}_a$ and the actual velocity $\boldsymbol{v}_a$. However, there is great flexibility in choosing different rules for how $\boldsymbol{u}_a$ is chosen. For instance, the direction of $\boldsymbol{u}_a$ could be coupled to the local average flow direction to achieve a herding effect (Helbing, Farkas & Vicsek, 2000). Or, the desired speed could be coupled to the relation between remaining distance to the preferred position and the time left before this position should be reached (Helbing & Johansson, 2009). In any case, the will force is inversely proportional to the relaxation time $\tau_a$ and is formulated as

$$\boldsymbol{f}_a^{\text{will}} = m_a \frac{\boldsymbol{u}_a - \boldsymbol{v}_a}{\tau_a}. \tag{3.9}$$

Note that if the agent is at rest, the will force is proportional to the desired velocity. This means that agents with a large desired speed can more easily force their way through a crowd than agents with a small desired speed.

### 3.1.4 Boundary forces

Interaction with boundaries of walls and other obstacles are treated in the same way as interaction between pairs of particles, both a social force and a contact force is applied. The position of an agent relative to a wall is given by the normal vector from the wall to the agent, the velocity of the wall is prescribed (usually zero), and the physical diameter of a boundary is assumed to be zero as well. In Helbing et al. (2002), it is mentioned that fire fronts are treated in a similar manner but with a larger interaction strength.

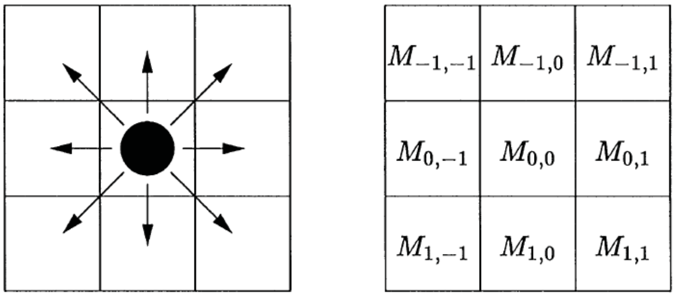


*Figure 3.2 An agent, its possible movements, and the associated matrix of preference, $\boldsymbol{M}_a$ (taken from Burstedde et al., 2001).*

## 3.2 The floor field cellular automata model

Cellular automata (CA) is a discrete, rule-based method. Two-dimensional models of pedestrian traffic based on the CA method were introduced more than a decade ago (Blue & Adler, 2000; Fukui & Ishibashi, 1999; Muramatsu & Nagatani, 2000). The floor field model (Burstedde et al., 2001; Kirchner & Schadschneider, 2002; Nishinari et al., 2004; Schadschneider, Kirchner & Nishinari, 2002) is by many considered the more flexible CA approach in that it combines mechanisms for finding the shortest free path to a target location with models of interactions with other agents and with infrastructure. This is achieved by taking inspiration from the motion of ants which is based on the process of *chemotaxis*, a chemical communication system (Schadschneider et al., 2009). In the two-dimensional floor field model, the underlying structure is a two-dimensional grid with a cell width which corresponds to the space occupied by a single agent, typically equal to 40 cm i both directions. All agents are assumed to have the same speed, and the size of the time step is chosen so that a moving agent moves exactly one cell width per time step. In this section we will first review the basic rules of the CA method, and then look at the generation of floor fields.

### 3.2.1 Basic rules

Each agent is given a direction of preference. From the preferred direction a $3 \times 3$ matrix of preference is constructed which describes the probability of an agent to move to one of the 9 nearest cells (see Fig. 3.2). Based on the resulting probability distribution, a desired move is drawn. This is done in parallel for all agents. If the target cell is occupied, the agent stays in the original cell. If the target cell is not occupied and no other agent targets the same cell, the move is performed. If more than one agent targets a given cell, only the agent with the higher relative probability will be allowed to perform the move. As a result, a cell can either be empty or contain exactly one agent. The basic matrix is modified by the effect of other agents and obstacles through the introduction of floor fields. In addition, rules for switching between two modes ("happy" and "unhappy") are introduced to prevent artificial jamming near obstacles.

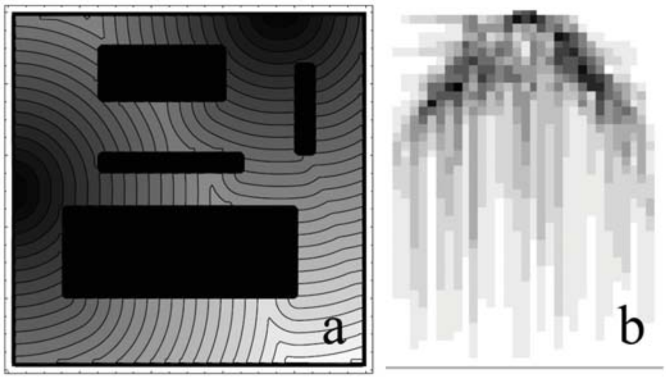


*Figure 3.3 Examples of floor fields in cellular automata models. Panel* **a** *shows a static floor field due to solid walls (taken from Nishinari et al., 2004), while panel* **b** *shows a dynamic floor field due to agents exiting a room through a door located in the middle of the upper boundary (taken from Schadschneider et al., 2009).*

### 3.2.2 Floor field generation

The basic movement probabilities are modified by two floor fields so that a movement in the direction of higher fields is preferred. The dynamic floor field, $D$, represents a virtual trace left by moving agents. This trace changes in time, not only due to the movement of the agents, but also due to processes of decay and diffusion which lead to the broadening and dilution of the trace in time. The static floor, $S$ does not change in time and reflects solid boundaries, doors etc. Examples of a static and dynamic floor field are shown in panels **a** and **b**, respectively, of Fig. 3.3. Taking into account the effect of the floor fields, the modified elements in the matrix of preference can be written as

$$p_{ij} = NM_{ij}e^{k_\mathrm{S}S_{ij}}e^{k_\mathrm{D}D_{ij}}(1 - n_{ij}), \tag{3.10}$$

where $N = 1/\sum_{ij} p_{ij}$ ensures normalization. The parameter $k_\mathrm{S}$ controls the influence of the static floor field and thereby determines the effective velocity of a single agent in the direction of its destination in the absence of other effects. The tendency of following in the footsteps of others, often called herding, is controlled by the parameter $k_\mathrm{D}$. The occupation number $n_{ij}$ is 0 for empty cells and 1 for occupied cells, thereby preventing movement to occupied cells.

## 3.3 FDS+Evac

The publicly and freely available FDS+Evac simulation software combines Fire Dynamics Simulator (FDS) (McGrattan & Forney, 2004), a computational fluid dynamics solver for modelling fire-driven fluid flow, with an agent-based egress calculation model (Hostikka et al., 2007; Korhonen et al., 2008a,b) to achieve a state-of-the-art fire simulation tool. As such, it represents a hybrid approach where elements of widely different computational techniques are used. In this section we will review the theoretical basis for the evacuation model. The discussion is split in 3 parts, the agent movement model, the interaction of agents and fire, and the model for agent

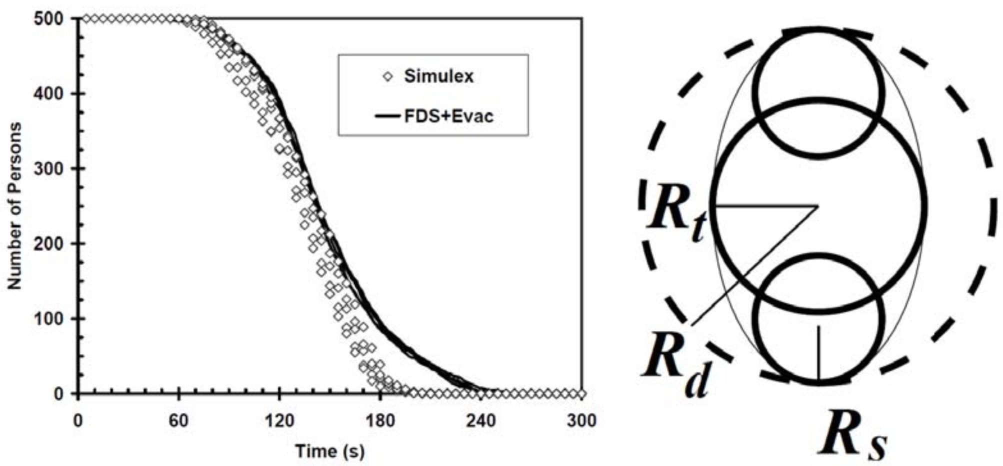


*Figure 3.4 Left panel shows a comparison of evacuation from a sports hall simulated with FDS+Evac (Korhonen et al., 2008b) and Simulex (Thompson & Marchant, 1995). Right panel illustrates the 3-circle agent model used in the FDS+Evac code.*

decision-making.

### 3.3.1 Agent dynamics model

The starting point for the agent dynamics model in FDS+Evac is the Helbing model described in section 3.1. This means that the movement of agents are primarily determined by social and physical forces acting on them. There are however a few extensions to the Helbing model implemented in FDS+Evac. First of all, the contact force has been extended relative to that given by Eq. 3.8 by the inclusion of a radial damping term, $c_d(\boldsymbol{v}_{ab} \cdot \hat{\boldsymbol{r}}_{ab})\hat{\boldsymbol{r}}_{ab}$. The parameter $c_d$ (typically $c_d = 500\text{kgs}^{-1}$) determines to what extent collisions between agents are considered inelastic. Secondly, the social interaction parameters are made velocity-dependent. In its simplest version, only the interaction strength, $A_a$, is varied. In the counterflow extension, both the interaction strength and the interaction range, $B_a$, is coupled to velocity, in addition to providing agents with rules on how to move around obstacles (Heilövaara, 2012). An illustration of the effect of extending the model to handle counterflow is seen in Fig. 3.5, where a single agent is meant to move through a crowd of agents moving in the opposite direction.

The agents are modelled by 3 circles rather than only one. The three-circle body model is taken from (Langston, Masling & Asmar, 2006) and is illustrated by the sketch on the right-hand side of Fig. 3.4. The three-circle body model is coupled to equations of rotation. The momentum equation for the rotation is

$$I_a^z \frac{d^2\phi_a(t)}{dt^2} = M_a^{\text{con}}(t) + M_a^{\text{soc}}(t) + M_a^{\text{will}}(t) + \eta_a^z(t), \tag{3.11}$$

where $I_a^z = 4.0\text{kgm}^2$ is the moment of inertia and $\eta_a^z(t)$ is a small random fluctuation torque (a similar random term is added to the momentum equation for translational movement). The deterministic contributions to the total torque due to physical contact, social repulsion, and will

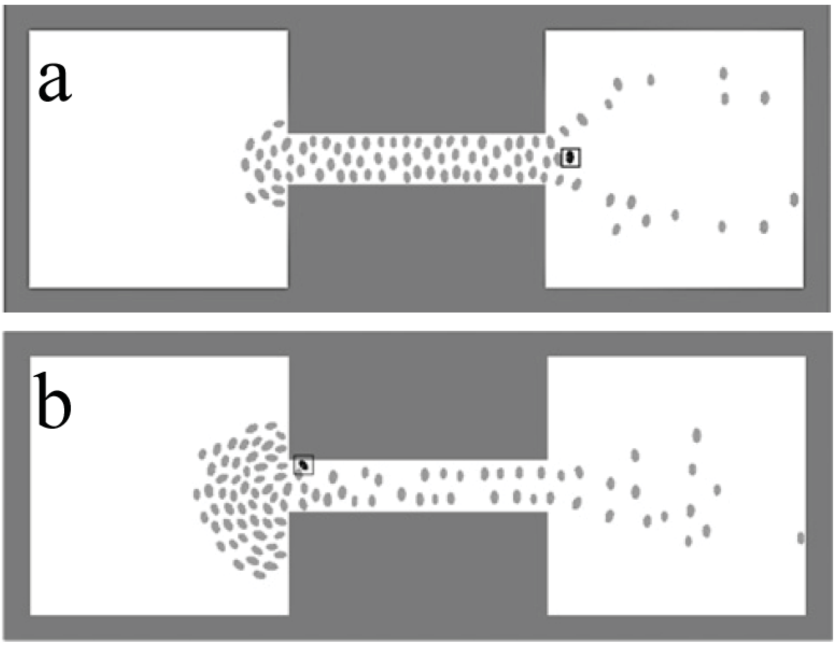


*Figure 3.5 Simulation of a single agent trying to move from right to left chamber through a 2-m wide corridor where 100 other agents are moving in the opposite direction. Panel* **a** *shows how the single agent fails in his attempt when the counterflow extension is not applied. In contrast, panel* **b** *shows that the single agent manages to move through the corridor, near the upper corridor wall, when the counterflow model is included (Heilövaara, 2012).*

forces are defined as

$$
\begin{aligned}
\boldsymbol{M}_a^{\text{con}} &= \boldsymbol{r}_{ab}^{\text{con}} \times \boldsymbol{f}_{ab}^{\text{con}}, && (3.12)\\
\boldsymbol{M}_a^{\text{soc}} &= \boldsymbol{r}_{ab}^{\text{soc}} \times \boldsymbol{f}_{ab}^{\text{soc}}, && (3.13)\\
M_a^{\text{will}} &= \frac{I_a^z}{\tau_a^z}[\varpi(t) - \omega(t)], && (3.14)\\
&\text{with} \\
\varpi(t) &= \frac{\phi_a(t) - \varphi_a}{\pi}\varpi_a^{\text{max}}. && (3.15)
\end{aligned}
$$

The spatial vector $\boldsymbol{r}_{ab}^{\text{con}}$ is defined as the vector from the centre of agent $a$ to the point of physical contact with neighbour $b$. Correspondingly, $\boldsymbol{r}_{ab}^{\text{soc}}$ is defined as the section of the inter-agent vector $\boldsymbol{r}_{ab}$ which starts at the centre of agent $a$ and ends at the surface of $a$. In the expression for the torque due to will forces, Eq. 3.14, $\tau_a^z$ denotes the rotational reaction time and $\omega(t) = d\phi_a(t)/dt$ is the angular velocity. The target angular speed, $\varpi_a(t)$ is shown in Eq. 3.15 to increase with increasing difference between the instant body angle ($\phi_a$) and the preferred body angle ($\varphi_a$). If we assume $-\pi \leq \phi_a - \varphi \leq \pi$, the absolute value of the target angular velocity is limited by $\varpi_a^{\text{max}} = 4\pi\text{s}^{-1}$.

### 3.3.2 Agent and fire interactions

From the Fire Dynamics Simulator, spatial information regarding gas temperature, smoke and gas densities, and radiation levels is obtained and can be used to modify the agent dynamics. So far, the focus has been on the effect of smoke on the speed of agents and on how toxic effects of the gas can incapacitate agents. The first effect has been studied experimentally (Frantzich & Nilsson, 2003). Based on this work, the preferred speed of agents is assumed to decrease linearly with increasing smoke concentration. The toxic effects of gaseous fire products are treated using Purser's Fractional Effective Dose (FED) concept (Purser, 1995). The present version of FDS+Evac considers only the concentration of CO, $CO_2$, and $O_2$. It is assumed that the concentration of $CO_2$ is sufficiently low to not have narcotic effects in itself but only have an effect due to its stimulating effect on hyperventilation. The total FED value is thus found as

$$\mathrm{FED}_{\mathrm{tot}} = \mathrm{FED}_{\mathrm{CO}} \times \mathrm{HV}_{\mathrm{CO_2}} + \mathrm{FED}_{\mathrm{O_2}}, \tag{3.16}$$

where the FED value due to CO is multiplied by the $CO_2$-dependent hyperventilation factor. An agent is considered incapacitated when the FED value exceeds unity. In this case, the agent becomes static but otherwise unchanged.

### 3.3.3 Model for agent decision-making

Decision-making in the context of evacuation simulations primarily deals with selecting an appropriate exit. Test versions of FDS+Evac have also implemented routines for simulating group behaviour but this is not available in the official version of the code (Korhonen et al., 2008b). Exit selection is done in a two-step process: First, all exits known to a given agent are sorted into 6 categories according to whether the exit is visible, familiar, and whether disturbing conditions such as increased temperature and smoke apply to the exit. The categories have been ranked based on: (i) Physical conditions at the exit, (ii) The agent's familiarity with the exit, and finally (iii) The visibility of the exit to the agent. If more than one exit is found in the top ranked category, the exit providing the fastest evacuation is selected. In estimating the evacuation time, the estimated time of queuing is added to the estimated time of walking.

## 3.4 Other models

In this section we will quickly review 4 other crowd dynamics models. The first 2 models focus on classic applications of pedestrian traffic, while the last 2 models are aimed at simulating the more complex scenarios related to riots.

### 3.4.1 Cognitive heuristics model

Due to what is perceived as limitations of other physics-inspired pair interaction models, Moussaïd, Helbing & Theraulaz (2011) propose a model of pedestrian behaviour based on a cognitive science approach. The model addresses the following two questions: (i) What kind of information is used by the pedestrian? And, (ii) how is this information processed to adapt the walking

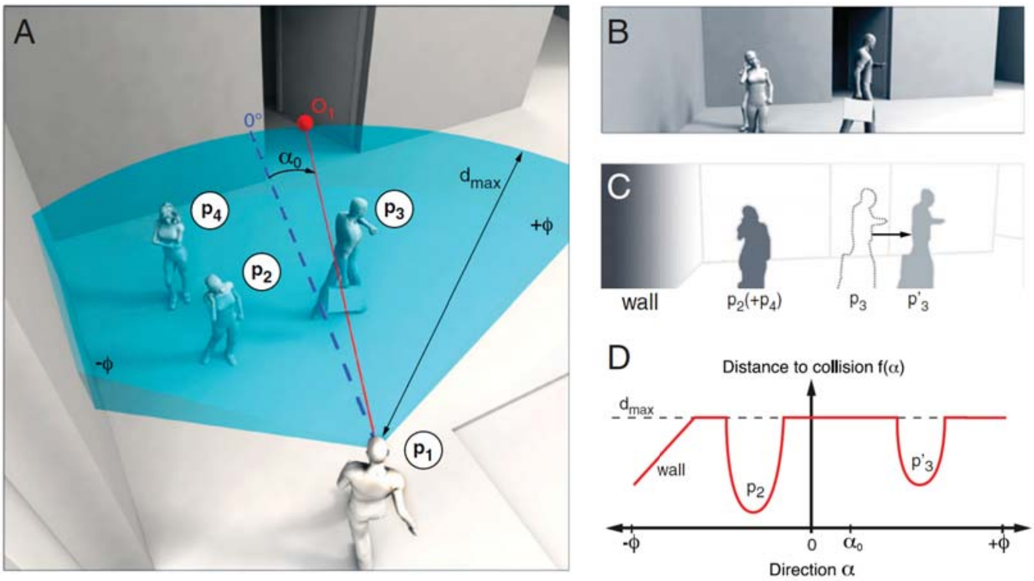


*Figure 3.6 Illustration of the cognitive heuristics model: Panel A shows a bird's eye view of an example where pedestrian $p_1$ try to reach destination point $O_1$ without walking into agents and walls found within the vision field (turquoise region). Panel B show the same scene from the perspective of agent $p_1$. Panel C represents an abstraction of the scene where darker areas correspond to shorter collision distances. Finally, panel D gives a graphical representation of the function $f(\alpha)$ reflecting the distance to collision as a function of the angle $\alpha$ (taken from Moussaïd, Helbing & Theraulaz, 2011).*

behaviour? Vision is in this case assumed to be the main source of information, and it is assumed that the preferred walking speed and direction is modified according to the visual information. Simulation results obtained with this method is shown to compare well with experimental data on pedestrian scenarios such as single agent manoeuvres in a corridor, unidirectional flow in a street at varying crowd density, and turbulent flows in front of a bottleneck (Moussaïd, Helbing & Theraulaz, 2011).

The locally preferred walking direction, $\alpha_{\text{des}}$, is found as a trade-off between avoiding obstacles such as neighbouring agents and minimizing detours from the most direct route to the destination point. The first heuristic of the model is therefore:

**A pedestrian $a$ chooses the direction $\alpha_{\text{des}}$ that allows the most direct path to destination point $O_a$, taking into account the presence of obstacles.**

Each agent has a field of vision which corresponds to an angle of $\pm\phi$ relative to line of sight. The chosen direction $\alpha_{\text{des}}$ is at any given point in time computed by minimizing the distance to the destination point in the direction $\alpha_0$ as:

$$d(\alpha) = d_{\text{max}}^2 + \text{f}(\alpha)^2 - 2d_{\text{max}}\text{f}(\alpha)\cos(\alpha_0 - \alpha), \tag{3.17}$$

where the function $\text{f}(\alpha)$ indicates the distance to the first collision as a function of angle. In computing $\mathrm{f}(\alpha)$, the agents take into account the other pedestrians' walking speeds and body sizes (representing the projection of an agent's body on the horizontal plane by a circle whose radius is proportional to the agent's mass). An upper limit to the allowed values of $\mathrm{f}(\alpha)$ is equal to $d_{\text{max}}$ and corresponds to the "horizon distance".

Because a time period $\tau$ is required for an agent to stop in case of an unexpected obstacle, agents should take this time delay into account when determining safe distances to other agents. The second heuristic of the model is therefore:

**A pedestrian maintains a distance from the first obstacle in the chosen walking direction that ensures a time to collision of at least $\tau$.**

For this reason, the instant preferred velocity becomes $v_{\text{des},a} = \min(v_a^0, d_h/\tau)$, where $d_h$ is the distance between pedestrian $a$ and the first obstacle in the preferred direction $\alpha_{\text{des},a}$. The desired velocity vector obtained with the cognitive heuristics model is then combined with the Helbing expression for the will force (see Eq. 3.9). To take into account body collisions in cases of overcrowding, physical contact forces with other agents and solid boundaries are included as in the Helbing model (see Eq. 3.8). The change in the actual velocity is determined by the momentum equation where the will force and contact force is included. Fig. 3.6 illustrates the principles of the method in a case where an agent, marked $\mathrm{p}_1$, tries to reach a door without colliding with 3 neighbouring agents and 3 walls found within the field of vision. Note that one of the agents, marked $\mathrm{p}_4$, is hidden behind one the other agents, and that the third agent, $\mathrm{p}_3$, is moving at roughly a right angle relative to the line of sight of agent $\mathrm{p}_1$.

### 3.4.2 PLEdestrians: A least-effort formulation

The least-effort formulation is based on the approximation that a person's caloric expenditure rate $R$ can be expressed as quadratic function of the velocity (Whittle, 2002). By integrating the expression for the caloric rate function over a given trajectory, we get an estimate of the total caloric cost of traversing the trajectory. This leads to the following expression for the energy $E_a$ required for an agent $a$ to move along a path $\Pi$:

$$E_a(\Pi) = m_a \sum_{\Pi} (e_{w,a} \boldsymbol{v}_a{}^2 + e_{s,a}) dt, \tag{3.18}$$

where $e_{w,a}$ and $e_{s,a}$ are constants depending on the gender, age, and fitness level of the agent. From Eq. 3.18 we can calculate an optimal speed (typically around 1.3 m/s for an average adult male). In a crowded environment, one must also take into account neighbouring agents and solid obstacles. This results in additional constraints on the optimization calculations. This leads to a two-step computational algorithm where, first, a set of permissible velocities an agent can take is determined, and secondly, a trajectory is chosen so as to minimize the caloric cost. (Guy et al., 2010, 2012). A snapshot from a full-scale pedestrian simulation using the least-effort formulation is compared with real-life photo in Fig 3.7.

The restrictions put on an agent's velocity to avoid collisions is somewhat similar to the cognitive


*Figure 3.7 Snapshot from a simulation using the least-effort formulation of a busy crossing at the Shibuya station in Tokyo, Japan (panel* **a***) compared with a still from a real-life video of the crossing (panel* **b***) (taken from Guy et al., 2010).*

heuristic model described in section 3.4.1: The constraint on the velocity of agent $a$ due to neighbouring agent $b$ is found by first computing , $\delta\tilde{\boldsymbol{v}}_{ab}$, the minimum change in the relative velocity of the two agents required to avoid a collision in the next $\tau$ seconds. This constrains the velocity of $a$ and $b$ to change by at least $\delta\tilde{\boldsymbol{v}}_{ab}/2$. The complete set of permissible velocities for a given agent is given by the union of the linear constraint from all neighbouring agents and nearby walls. With the contraint that the agent is limited to paths whose initial velocity lies within the set of non-colliding velocities, Eq. 3.18 is minimized with respect to caloric cost. As a simplifying assumption, the method is restricted to paths $\Pi$ that can be represented by two linear segments: The first segment corresponds to a motion to avoid collisions with nearby obstacles, and the second segment leads the agent directly towards its target position. The least-effort formulation has been validated against standard pedestrian scenarios

### 3.4.3 The Epstein civil violence model

The models described so far have either represented normal behaviour scenarios such as pedestrian traffic applications or evacuation scenarios where the motivations and goals of agents are clearly defined. Now, we will look at a model which focuses on the dynamics in a potentially violent crowd. The Epstein civil violence model comes in two different versions. One deals with the scenario of a generalized rebellion against central authority, the other is directed towards inter-group violence (Epstein, 2002). In the following discussion, we will focus on the former version.

The model involves two categories of actors: **Agents** are members of the general population which might be actively rebellious or not. **Cops** are the forces of the central authority, whose task is to arrest actively rebellious agents. Associated with each agent are 5 socio-political parameters, level of risk aversion ($R$), perceived hardship ($H$), perceived legitimacy of the regime ($L$), level of grievance ($G$), and the agent's activity state ($S$). The first 3 parameters are randomly set for each agent from a uniform distribution between 0 and 1. Grievance is related to hardship and legitimacy through the simple relation:

$$G_a = H_a(1 - L_a). \tag{3.19}$$

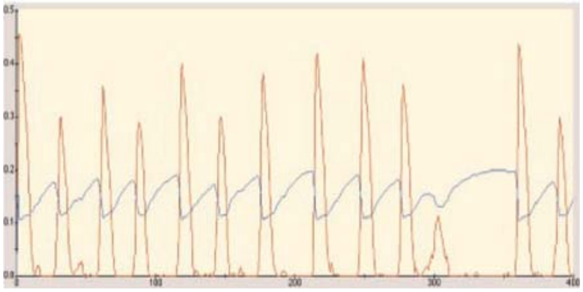

*Figure 3.8 Group-averaged tension (blue curve) and rebellious activity level (red curve) as function of time in a simulation rebellion dynamics (taken from Epstein, 2002).*

Eq. 3.19 tells us that grievance becomes large if hardship is large and legitimacy is low. A large grievance value is the basis for active rebellion. However, most people would also consider the risk of being arrested before actually becoming an active rebel. The risk of being arrested, as assessed by a given agent $a$, is coupled to the ratio of cops to active rebels within the field of vision of agent $a$. The field of vision is defined as a certain area centred on the position of the agent. The arrest probability is given as

$$P_a = 1 - e^{-kN_a^C/N_a^A}, \tag{3.20}$$

where the constant $k$ is chosen so as to provide a suitable probability when $N_a^C = N_a^A$. The integers $N_a^C$ and $N_a^A$ denote the number of cops and agents, respectively, within the field of vision. Note that $N_a^A \geq 1$ because agent $a$ always counts herself as being a potentially active rebel. The net risk can be defined $N_{\text{risk},a} = R_a \cdot P_a$ and $T_{\text{risk}}$ is defined as non-negative threshold risk. If $G_a - N_{\text{risk},a} > T_{\text{risk}}$ then agent $a$ becomes an active rebel. If not, agent $a$ becomes quiet.

The cops have a clearly defined task every time step: To arrest a randomly chosen active agent found within the field of vision. The field of vision of the cops and agents need not be the same. The rule for spatial movement of both agents and cops is highly simplistic: Each time steps, all agents and cops move to a random site within their own field of vision. Based on these simple rules, the model can model how active rebellion in a group of people can vary in time. An interesting aspect of this model is how one can monitor the build up of tension in the group. If the average grievance is high, but the average level of rebellious activity low because the typical risk aversion is also high, then the tension could be considered high. In Fig. 3.8, the relation between tension and the rebellious activity level in the model is illustrated. The graph shows two curves plotted as functions of time. The red and blue curves correspond to the group-averaged tension level and activity level. Most notably, is the way a tension build-up is followed by an outburst of high activity level and a shock-like drop in tension. An empirical study later found the explanatory power of the model to be high (Klemens et al., 2010).

### 3.4.4 The Jager approach-avoidance model

As an example of a rule-based simulation model, we will briefly review the Jager approach-avoidance model developed in order to study clustering and fighting in two-party crowds (Jager,

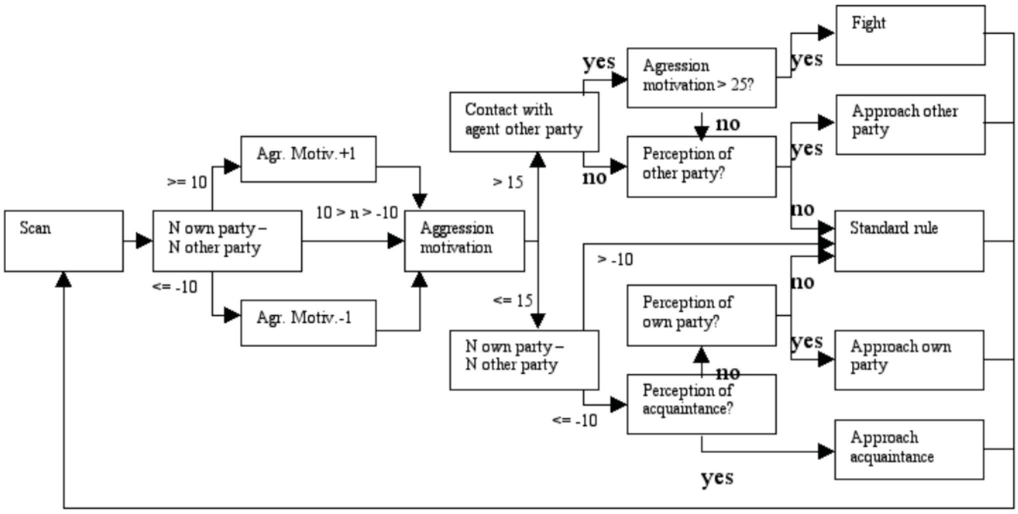


*Figure 3.9 Graphic representation of the behavioural model in the Jager model which illustrates how the agent behaviour is a function of the* **aggression motivation** *of the agent and the local distribution of neighbouring agents, both of own and other party (taken from Jager, Popping & van de Sande, 2001).*

Popping & van de Sande, 2001). The model is based on the assumptions that the tendency to approach or to avoid is a fundamental behavioural characteristic and the that behaviour of agents is not strictly deterministic. All simulations are restricted to a $100 \times 100$ grid where only one agent can occupy a single cell. In other words, this model is in the tradition of cellular automata (see section 3.2). Similarly, the time step is fixed to 1s, and the agents can during a given time step be static or move to a neighbouring cell. The crowd forming a party are heterogeneous and is typically split into groups of friends or acquaintances.

The model further acknowledges observational work which states that some 90% of the people present at riot situations are quite calm and only fulfil the role of spectators. For this reason, agents belong to one of the three types **hard-core** rioter, **hangers-on**, or **spectators**. The difference between the behaviour of the three agent types, is simply how often the agents scan the surrounding area. A hard-core rioter scans the area twice as often as a hanger-on and 8 times as often as the spectators. Fig. 3.9 graphically explains how the scanning process can lead to behavioural changes for a given agent $a$: Let $\delta N_a = N_{\text{own},a} - N_{\text{other},a}$ denote the difference between number of neighbouring agents belonging to the agent's own party and the number of neighbouring agents belonging to the other party. If $\delta N_a \geq 10$, then the level of **aggression motivation** for agent $a$, let us refer to it as $M_a$, will increase by 1. If on the other hand $\delta N_a \leq -10$, then $M_a$, will decrease by 1. If $15 < M_a \leq 30$ the behaviour will be more offensive or even aggressive. In the latter case, agent $a$ will engage a neighbouring agent from the other party in a fight lasting for 100 s. If $M_a \leq 15$ the behaviour of agent will be neutral or possibly defensive. In the latter case, this means approaching agents of own party, choosing acquaintances if this is possible.

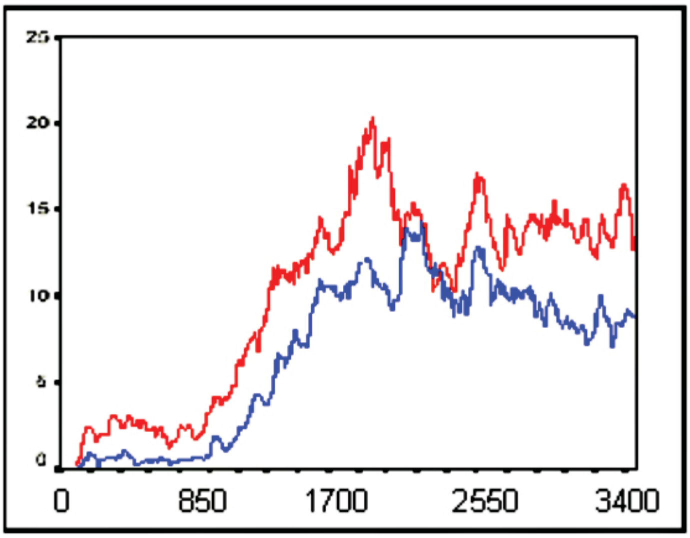


*Figure 3.10 Number of on-going fights as a function of time in a crowd of 400 agents where the relative size of the two parties is 3:1 and where the proportion of hard-core rioters is either 5% (red curve) or 1% (blue curve) (Jager, Popping & van de Sande, 2001).*

The model has three independent variables. These are the size of the crowd, the relative size of the two parties, and the proportion of hard-core individuals. It is assumed that the number of hangers-on is twice the number of hard-core agents. Fig. 3.10 shows results from a simulation where 400 agents in total are included. The relative size of the two parties is 3:1, and the percentage of hard-core agents is either 5% (red curve) or 1% (blue curve). The graph shows the number of on-going fights as function of time. After about 15 minutes, the number of fights increases quite steadily until a peak is reached at around half an hour. The peak is almost twice as large when the hard-core agent percentage is 5% rather than 1%. At later times, when the system has gone through a linear phase and a saturation phase, the number of fights is stabilizing at a lower level. It might seem like the time scale in the stabilizing phase is slower in the case of the lower hard-core agent percentage.

### 3.5 Commercial crowd modelling software

There are a number of commercial crowd modelling software packages available aimed at a wide range of applications, from digital entertainment to architectural design and crowd management training. However, the scientific value of such software is often difficult to assess since (i) technical details regarding the software are usually confidential in order to secure commercial interests, and (ii) emphasize is often put on crowd animation rather than crowd simulation. Nevertheless, we will briefly mention a handful of commercially available crowd modelling software packages in this section. The information put together here is taken from the web sites of the individual software companies and from the review in Challenger, Clegg & Robinson (2009).

**Legion:** Legion combines two-dimensional simulations with two- or three-dimensional visualization. The agents move through the environment according to the principle of least effort, with minimal time, minimal costs (dissatisfaction and discomfort), minimal congestion and maximum speed (Still, 2000). The agents also have the ability to make decisions regarding which route to take, considering environmental circumstances such as queuing. Properties of the agents, such

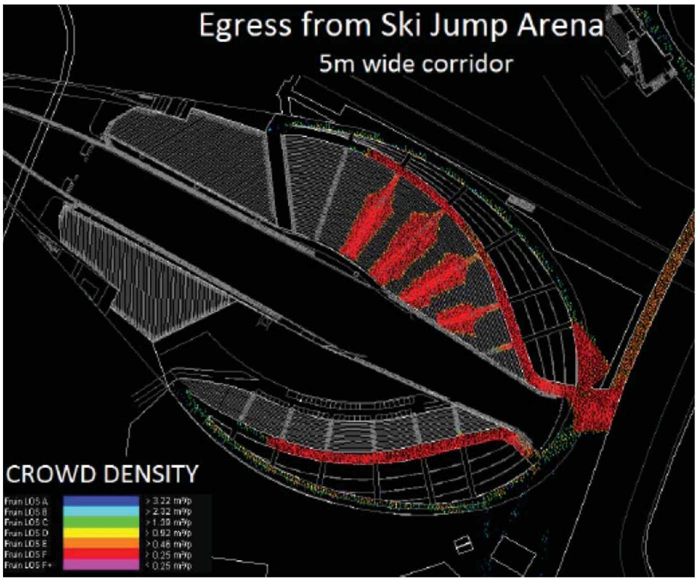


*Figure 3.11 Simulation of egress from Holmenkollen ski jump arena. The plot shows crowd density as a function of position. The analysis was performed by Movement Strategies AS (Movement Strategies AS, 2012).*

as size, preferred speed, age, and amount of luggage, can be assigned randomly from a suitable distribution. The software is primarily directed towards applications of pedestrian traffic, evacuation processes, and planning of large events. Fig. 3.11 shows an example of the use of Legion in studying egress from a ski jump area (Movement Strategies AS, 2012).

**Myriad II:** Just as with Legion, Myriad II is based on the research of Prof. Keith Still. The software combines methods for network analysis, spatial analysis, and agent-based analysis into one modelling suite. This makes it possible to study part of the problem using a simpler approach such as by network analysis, and restrict the use of agent-based analysis to areas characterized by more complex interactions. As in Legion, the agent dynamics in Myriad II is calculated using the least effort principle. In this case, the software is designed to test boundary conditions rather than specific circumstances, This means assessing the effect of difference in flow rate, density, ingress, circulation, and egress. The agents are capable of scanning, seeing, and reacting to the environment.

**MassMotion:** According to the user guide, MassMotion has been developed to be a generalized pedestrian simulator. The space is represented by three-dimensional geometry. The calculation of crowd movement is separated into two distinct processes. The first component governs the individual agents' basic movements and and how they respond to changes in the environment.This is done using a modified social force algorithm. The second component is concerned with network path planning between origins and destinations. It analyzes distance, congestion, and terrain to develop costs for all available routes to the agent goal and to select an appropriate cost-effective route.

**Massive:** Massive is well-known as a crowd simulation software, primarily due to its use in digital entertainment. This is understandable as it was developed for the film trilogy **Lord of the Rings**. Since then, it has become an industry standard in digital film production. In later years, the area of application for the code has been extended to include engineering simulation and architectural visualization. The agent dynamics is controlled by fuzzy logic in combination with a library of pre-programmed manoeuvres which makes the agents respond to sight, hearing, and touch.

# 4 Empirical Studies

So far in this report, we have reviewed a wide range of different approaches to numerical modelling of crowd dynamics. But no matter what approach is chosen, the computational models can only to a certain degree be founded on strict logic. In addition, the models must rely on assumptions made regarding human behaviour. It is therefore of great importance to calibrate the computational models against empirical data. In this section we will review a handful of the most important empirical work on crowd dynamics. We first focus on pedestrian dynamics, or crowd dynamics under normal conditions. After that, we review empirical work on evacuation, riots, and crowd disasters.

## 4.1 Pedestrian dynamics

As mentioned in the introduction, empirical studies of pedestrian flow started to emerge in the late 50s and in the 60s (Hankin & Wright, 1958; Navin & Wheeler, 1968; Oeding, 1963; Older, 1968). Right from the start, this has been studied from an engineering point of view, focusing on improving pedestrian traffic (Fruin, 1971; Predtechenskii & Milinskii, 1978). The most frequently quoted studies of this type are probably the studies of Fruin (1971) and Weidmann (1992).

Fruin introduced the concept of level-of-service standards (LOS) as a measure that defines relative degrees of convenience for different pedestrian traffic volumes and densities. Table 4.1 describes the 6 LOS categories and corresponding levels of flux and inverse density. Fruin also described time-space analysis where the product of available space and available time is compared to the product of the number of people passing through the area and the time it takes for them to pass through the area. He also looked at human body dimensions, locomotion characteristics (walking), and behavioural preferences.

Weidmann took the empirical study of pedestrian flow to a new level with his thesis (Weidmann, 1992). He collected information regarding step length and rate of energy consumption associated with pedestrian motion (see Fig. 4.1). He also derived the following emprically founded expression for pedestrian velocity $v$ as function of density $\rho$ given a certain preferred velocity $u$:

$$v = u\left(1 - \exp\left[-\gamma_W\left(\frac{1}{\rho} - \frac{1}{\rho_{\text{max}}}\right)\right]\right), \tag{4.1}$$

where $\rho_{\text{max}}$ should correspond to a maximum density and $\gamma_W$ is a density constant. Fig. 4.2 shows how this parameterized model fits with empirical data for $u = 1.34$ m/s, $\rho_{\text{max}} = 5.4$ agents/m$^2$, and $\gamma_W = 1.913$ agents/m$^2$.

Although the work of Fruin and Weidmann are qualitatively in agreement with each other, there are quantitative discrepancies between the two. Later studies have tried to reduce the uncertainties by combining information from several studies with new experimental data (Kretz, Grünebohm

| LOS | $J$ [1/(ms)] | $1/\rho$ [m$^2$] | Description |
|---|---|---|---|
| A | $\leq 0.4$ | $\geq 3.3$ | Threshold of free flow, convenient passing, conflicts avoidable. |
| B | 0.4-0.55 | 2.3-3.3 | Minor conflicts, passing and speed restrictions. |
| C | 0.55-0.8 | 1.4-2.3 | Crowded but fluid movement, passing restricted, cross and reverse flows difficult. |
| D | 0.8-1.1 | 0.9-1.4 | Significant conflicts, passing and speed restrictions, intermittent shuffling. |
| E | 1.1-1.4 | 0.5-0.9 | Reverse, passing and cross flows very difficult; intermittent stopping. |
| F | - | $\leq 0.5$ | Critical density, flow sporadic, frequent stops, contacts with others. |

*Table 4.1 Walkway level-of-service (LOS) categories with corresponding levels of pedestrian flux ($J$) and inverse density ($1/\rho$).*

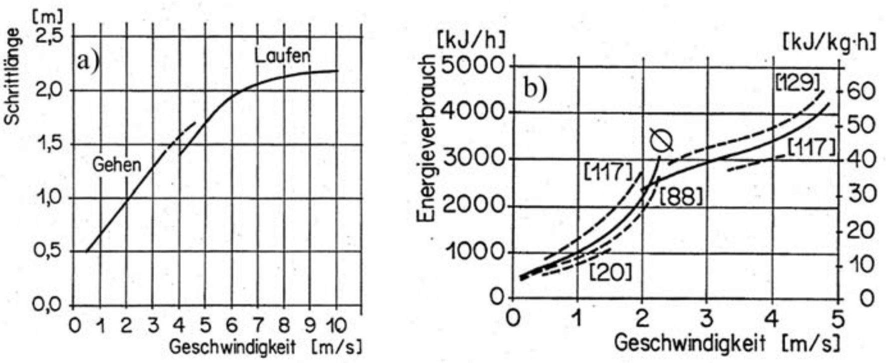


*Figure 4.1 Step length during pedestrian movement (panel **a**) and rate of energy expenditure as a function of velocity (panel **b**) (Weidmann, 1992).*

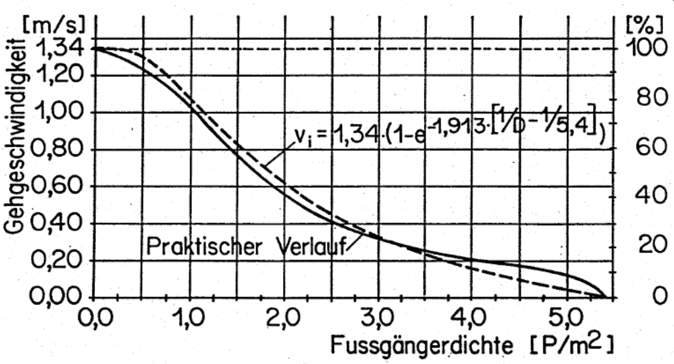


*Figure 4.2 Relation between pedestrian density and velocity as found directly from empirical data (solid line) and the parameterized model from Eq. 4.1 (Weidmann, 1992).*

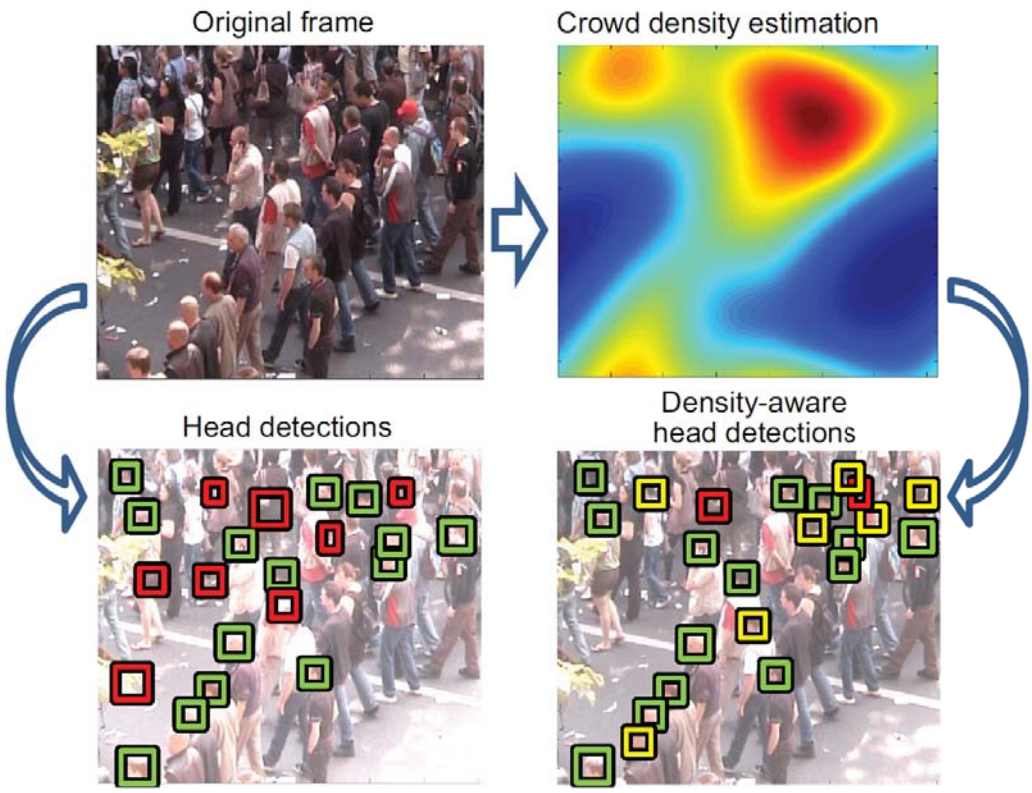


*Figure 4.3 Comparison of standard and density-based head detection algorithms: (i) The standard algorithm applies the object detector directly on the original snapshot (lower left panel), while (ii) the density-based algorithm combines information on the estimated crowd density with an object detector (lower right panel). Red and green squares indicate false and true positives, respectively, with both methods. Yellow squares indicate true positives found only using the density-based approach. The figure is taken from Rodriquez et al. (2011).*

& Schreckenberg, 2006; Moussaïd et al., 2012; Schadschneider et al., 2009). Others perform experiments specifically with simulations in mind. In other words, data from experiments are used directly to optimize parameters in a specific numerical model (Johansson, Helbing & Shukla; Scovanner & Tappen, 2009).

Evaluation of real-world crowds is another important area of empirical research. Here, a main challenge has been to automatically extract data from video footage using pattern recognition techniques. An example of an advanced person detection scheme utilizes crowd density estimates to improve the detection algorithm (Rodriquez et al., 2011). The detection scheme is illustrated in Fig. 4.3. Starting from the original snapshot (upper left panel), a state-of-the-art object detector (Felzenszwalb, 2010) can be used to identify heads in a crowd. However, due to the complexity of information in a picture of a reasonably dense crowd, the standard detection scheme results in a relatively small number of true positives and a number of false positives (green and red squares, respectively, in lower left panel). By estimating the crowd density in the picture (upper right panel) and feeding the density information into the object detector, the number of true positives is increased by 50% and the number of false positives is decreased by 75% (yellow and red squares, respectively, in lower right panel).

### 4.2 Evacuation, riots, and crowd disasters

The study of pedestrian dynamics is expected to provide insight into the normal dynamic state of a human crowd, a state where psychological and social aspects of the human interaction are reasonably well understood. In contrast, events of evacuation and riots represent exceptional conditions where many aspects of human nature can be of importance in predicting characteristics of the crowd dynamics. In the extreme, exceptional crowding can lead to crowd disasters. Since events of this type are strongly dependent on social and psychological factors, in addition to the purely physical factors, it is a major challenge to study the underlying mechanisms. Not only is it difficult to arrive at a constructive simulation model. It is equally difficult to design controlled experiments which adequately take into account the effects of fear, anger, and other strong emotions, which may or may not play a role. For this reason, the study of exceptional crowding events has largely been restricted to descriptions of real events, or collecting "anecdotal evidence" (Dickie, 1995; Elliott & Smith, 1993; Helbing & Mukerji, 2012). Qualitative descriptions of such events have in the past been coloured by political and social views of the observer. In addition, the information regarding an event is often both limited and biased (Helbing, 2010).

Limitations in the descriptive approach are likely to have contributed to the formation of the **7 myths of the traditional foundation of crowd research**. These theories have largely been rejected in modern day crowd research as stereotypes (Couch, 1968). We here review the myths as listed by Wijermans (2011):

1. Myth of irrationality: The idea that individuals in a crowd lose rational thought. Available evidence supports the opposite idea, that individuals largely act rationally given the

information and goals they have (Adang, 1998; Couch, 1968).

2. Myth of emotionality: The idea that individuals in a crowd become more emotional. Although crowd phenomenon often are associated with settings where strong emotions are present, high level of emotions is not a result of the crowding.
3. Myth of suggestibility: The idea that individuals in a crowd are more likely to obey or imitate. Some models include imitation as a separate mechanism (Baron & Kerr, 2003), while others claim that behaviour which might resemble imitation in reality is caused by other effects (Wijermans, 2011).
4. Myth of destructiveness: The idea that individuals in a crowd are more likely to act violently. Crowds are not generally associated with a higher level of violence (Adang, 1998; Couch, 1968). The reason for individuals to act violently is event specific and not inherently linked to the crowd setting.
5. Myth of spontaneity: The idea that violence occurs more suddenly in a crowd. This is a combination of the myths of irrationality and destructiveness.
6. Myth of anonymity: The idea that individuals in a crowd feel more anonymous. Experimental studies contradict the deindividuation theories claiming behaviour in crowd to be less controlled by social norms (Postmes, Spears & Lea, 1998). Again, event-specific parameters can be such that the social influence from peers can be stronger than the social norms formulated in laws etc.
7. Myth of uniformity: The idea that all individuals in a crowd act in the same way. Studies show that people in a crowd behave differently and that for instance only a small fraction of the people present at a riot actually are involved in violent activities (Adang, 1998).

Generally, the myths can be viewed as over-simplified or wrongly interpreted observational features of crowds associated with some types of exceptional events, such as for instance a riot.

### 4.2.1 Evacuation

Evacuation events are closely linked to pedestrian traffic in that the fundamental process is transportation of pedestrians. Therefore, several experimental studies treat evacuation events as cases of pedestrian flow with little attention to social or psychological aspects (Fang et al., 2010; Lei et al., 2012) of the evacuation. Others have tried to distinguish between normal flow, controlled (non-competitive) evacuation and panic (competitive) evacuation (Was, 2010). This separation was to some degree achieved by instructing the test subjects to walk normally, evacuate while optimizing the evacuation time for the whole group, or evacuate with the aim of optimizing their own, personal evacuation time. The role of visibility in an evacuation event has been studied by blindfolding test subjects (Isobe, Helbing & Nagatani, 2004). Animal experiments have also been performed to gain insight into dynamics of panic evacuation (Saloma et al., 2003). However in the end, panic evacuation in a human crowd can only fully be explored empirically by studying real-life, dramatic evacuation events such as the 1993 bombing of the World Trade Center (Aguirre, Wenger & Vigo, 1998).

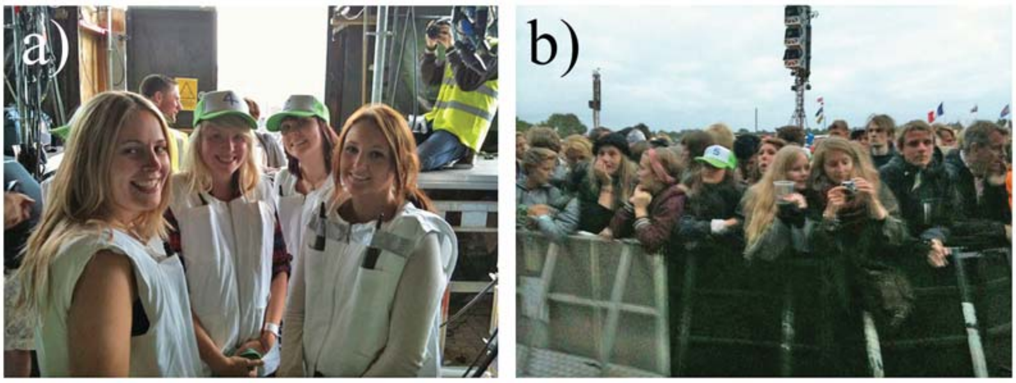


*Figure 4.4 Pressure suits, 4 of in all 6, worn at the Roskilde Festival 2011 (panel* **a***). The suits worn by the test subjects measured pressure levels as function of time in the crowd in front of the stage (panel* **b***). The images are taken from Still (2011).*

### 4.2.2 Riots and large gatherings

What distinguishes riots and gatherings of large crowds from scenarios of pedestrian dynamics, is that not all phases of an event is dominated by systematic transportation of individuals in the crowd. True enough, transportation of people at the start and (in particular) at the end of events such as a demonstration or a rock concert are often the most critical phases of the event (Still, 2000). However, crowd events of this type also contain a phase where people are more or less motivated to stay roughly in the same area for a certain period of time. Whether the event can be classified as a gathering, a demonstration, or a riot, depends on form and content of individual and collective behaviour during the interim phase when a number of people are in the same location at the same time (McPhail, 1983). For this reason, motivations and intentions of the individuals in the crowd can be expected to play an important role. Klemens et al. (2010) studied empirically the conditions for rebellious events in light of the theoretical model of Epstein (2002). Others have looked at the social-psychological interactions during a riot, e.g. in connection with sports events (Russel, 2004). It is also of interest to find experimental evidence for the level-of-service model of Fruin (1971) by estimating densities in real crowds (Weppner & Lukowicz, 2011). In the case of extremely high densities ($5-7\ \mathrm{m}^{-2}$), we know from damage on railings and other infrastructure that pressure levels due to pushing in fatal events have exceeded 4500N (Fruin, 1993). Experiments to determine forces on guardrails due to leaning and pushing have shown that force levels of 30% to 75% of participants weight can occur (Fruin, 1993). Fig. 4.4 shows two images taken at a pressure suit experiment at the Roskilde Festival 2011. Similar tests have been performed at Wembley stadium in 2009 and at the Roskilde Festival in 2010 (Kemp & Coole, 2010).

## 5 Concluding remarks

In this report I have presented a review of available research on crowd modelling. I have also to some degree included literature on relevant empirical work. Although the aim of the study is to gain insight into the modelling of exceptional crowd events such as demonstrations and riots, a large portion of the described research deals with normal state, pedestrian traffic. This is partly due to the fact that the open literature on crowd modelling is dominated by this particular class of applications, but also because treating normal behaviour crowd dynamics is a necessary first step in modelling also the more extreme cases of crowd dynamics. Still, it should be emphasized that crowd dynamics in exceptional scenarios is expected to be more challenging to model than normal state scenarios due to the increased influence of complex social and psychological factors.

Based on the research described in this review, a number of conclusions can be made: First of all, it seems fair to say that crowd dynamics is well established as an important topic in urban planning and safety and security engineering. This is illustrated by the commercial tools that are available which effectively applies crowd dynamical principles to relevant problems. At the same time, crowd dynamics is an interesting and constantly evolving area of scientific research. The main challenge in gaining new insight into the field of crowd dynamics, is probably the fact that this research is highly interdisciplinary. This makes it inherently difficult to fully utilize all relevant literature. Researchers will generally relate more easily to that part of existing research which fits better with their own research background, be it sociology, psychology, or physics. Furthermore, any researcher will try to contribute by applying his or her own expertise to the problem at hand. This is natural. The only danger is, that contributions stemming from other scientific disciplines are more easily rejected by the researcher. As a result, it becomes difficult to merge findings produced with widely different research approaches. Communication across disciplines is difficult not only because the scientific language changes from one discipline to another, but also because fundamental scientific perspectives are different in different disciplines. In addition, the lack of communication across disciplines is self-reinforcing because the less knowledge we have on a scientific topic, the harder it is to understand the research on this topic.

Another striking feature of crowd dynamics, is the importance of crowd density on the crowd behaviour. In the low density regime, people are acting completely independent of each other and collective behaviour is non-existent. The actions of individuals are controlled by the scenario specific settings and the motivations and plans that individuals have in a given scenario. If this is the case, then clearly we cannot predict deterministically people’s behaviour in any given scenario. An approach based solely on natural sciences most likely would not be particularly fruitful, while psychological effects are probably very important for the outcome. However as crowd density increases, effects of collective behaviour start to emerge. These effects are in many ways similar to what one can observe in herds of animals or even in ensembles of non-living, self-propelling particles. This tells us that fundamental laws of physics are probably starting to become important to the understanding of the crowd dynamics, at the expense of psychological and social effects. In the case of extremely high crowd density, there is no longer any room for individuals to make intelligent decisions that affect the dynamics. In this regime, I believe it is safe to assume that only physical forces play an important role. In most realistic scenarios involving human crowds, the crowd density is on some intermediate level. In this case, psychological, social, and physicical effects will all play an important role in determining the dynamics of the crowd.

Two different but similar sized crowds will never behave exactly the same in a given scenario. It is therefore futile to search for a crowd model that can predict correctly the behaviour of any crowd in any given setting. However, one can hope that future models are more successful at incorporating knowledge from different disciplines, and that these models provide a better balance between different effects that are important in making statistically accurate predictions on crowd behaviour.

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