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A MODEL OF INTERNAL BALLISTICS PROPERTIES IN GUN TUBES

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8) ABSTRACT

In this report a study of the pressure generated by burning of gunpowder in gun barrels has been carried out. Calculations using the model were compared with experimental results and with results from the standard NATO internal ballistic code (IBHVG98).

The numerical calculations gave good agreement with experiments of the gas pressure in gun barrels and of the velocity of projectiles. This indicates that the implemented model is viable and can be used for instance in parametric studies. By using the model the scattering of some experimental results are analyzed. Also some critical physical assumptions in the IBHVG98 code are pinpointed.

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1 INTRODUCTION

During 2001 a contact between FFI and the Norwegian Army was signed. The objectives were to analyse in detail some of the mal-functions of the gun, which has been observed during launching of projectiles. Of interest were mal-functions caused by leakage of the gunpowder gases during launching. To support the numerical analysis, calculations by using a constructed mathematical model of the internal ballistic properties has been performed. Of special interest was to construct a code that can handle different forms of equations of states at large pressures up to 1.5 GPa. The traditional Nobel-Abel equations of state used in most ballistics codes is not good enough at such large pressures. Pressures of this range can be reached when for instance, a) the projectiles ignites during launching, b) the projectile is deadlocked inside the gun barrel implying large pressures build-up of the gunpowder gases. Also, critical assumptions of the model are pinpointed and compared with assumptions in the standard IBHVG98 code. A special issue was to analyze the mathematical model descriptive of the heat flux into the gun barrel.

The numerical calculations gave good agreement with the experimental results of the gas pressure in the gun barrel and of the exit velocity of the projectile

2 THE BURNING MODEL

Consider as an example the chemical reaction as burning of identical particles where the burn rate of a surface is given as

$$\dot{b}(t) \overset{\text{mod}}{=} H(\beta p_g), where\ \beta \overset{\text{def}}{=} 1/Pa \tag{2.1}$$

where "mod" means model assumption, and "def" means a definition, and where

$\dot{b}(t)$: Velocity of a burning front

H( ): Burn rate function

p_g : Gas pressure outside the surface particles (in the pores)

H( ), the burn rate function, is typically dependent of the gas pressure. Let a number of N identical particles burn inside a volume V. A burn fraction $F(t)$ (reaction ratio) is defined as

$$\begin{aligned} F(t) &\overset{\text{def}}{=} (M_s(t_0) - M_s(t))/M_s(t_0) = 1 - M_s(t)/M_s(t_0), \\ M_s(t) + M_g(t) &= const. = M = M_s(t_0) + M_g(t_0), \end{aligned} \tag{2.2}$$

where

$M_s(t_0)$: Total amount of solid initially in the volume V
$M_s(t)$: Total amount of solid in the volume V at time t
$M_g(t)$: Total amount of gas in the volume V at time t
$M_g(t_0)$: Total amount of gas initially in the volume V
M : Total amount of gas and solid in the volume V

Typically of 12.7mm guns, $M_s(t_0) = 1.52\,10^{-2}\,kg$. Assume as an example that the particles are cylindrical with one hole in the middle. Let R_1 and R_2 be the initial inner and outer radius of the surface, respectively. Let L be the initial length of the particles. Typical values of the gunpowder are that $L = 1.88\,10^{-3}\,m$, $R_1 = 8.00\,10^{-5}\,m$, and an outer surface radius of $R_2 = 7.00\,10^{-4}\,m$. It follows from (2.1) and (2.2) that

$$1 - F(t) = M_s(t)/M_s(t_0) = (r_2(t)^2 - r_1(t)^2)l(t)/[(R_2{}^2 - R_1{}^2)L] \qquad (2.3)$$

where

$l(t)$: The length of a cylindrical particle at time t
$r_2(t)$: The radius of the outer burning surface at time t
$r_1(t)$: The radius of the inner burning surface at time t

Assuming that all surfaces burn with the same velocity, gives that

$$l(t) = L - 2\,b(t), r_2(t) = R_2 - b(t), r_1(t) = R_1 + b(t) \qquad (2.4)$$

Inserting (2.4) into (2.3) gives that

$$F(t) = 1 - [R_2 - R_1 - 2\,b(t)][L - 2\,b(t)]/[(R_2 - R_1)L], b(t) \le Min[L/2, (R_2 - R_1)/2] \qquad (2.5)$$

Gun powder typically burn as [1]

$$\dot{b}(t) \overset{\text{mod}}{=} 1.1\;10^{-9}(\beta p_g)m/s,\; \beta = 1/Pa \qquad (2.6)$$

In a volume V there exists an amount of gas and an amount of solid. The following densities are defined

$$\rho_g \overset{\text{def}}{=} M_g/V_g, \rho_s \overset{\text{def}}{=} M_s/V, \rho \overset{\text{def}}{=} (M_s + M_g)/V, V = V_g + V_s \qquad (2.7)$$

where

ρ_g : Density of gas in the total volume V
ρ_s : Density of solid in the total volume V
V_g : Volume of gas in the total volume V
V_s : Volume of solid in the total volume V

Observe the asymmetry in the definitions of the gas density and the solid density. It now follows from (2.2) and (2.7) that

$$\rho_s = M_s(t)/V = (1-F)M_s(t_0)/V = (1-F)(M - M_g(t_0))/V = (1-F)\rho - (1-F)M_g(t_0)/V$$
$$\rho_g = M_g(t)/V_g = \frac{M - M_s(t)}{V[1 - V_s/V]} = \frac{\rho F + (1-F)M_g(t_0)/V}{\{1 - [(1-F)\rho - (1-F)M_g(t_0)/V]/\rho_{pa}(\overline{p})\}} \tag{2.8}$$

where

$\overline{p}$: Average density , defined later.
$\rho_{pa}(\overline{p}) \overset{\text{def}}{=} M_s / V_s(\overline{p})$: Density of solid particles

The external function $\rho_{pa}(\overline{p})$ gives the compression of the solid particles due to the gases and to the interaction with other solid particles. If the particles are stiff, the relation is simply that $\rho_{pa}(\overline{p})$=constant, which we assume in this article. Typically $\rho_{pa}(\overline{p}) = 1.60\,10^3 kg/m^3$.

Let $v_p(t)$ be the velocity of the rear of the projectile during motion in the gun barrel. The equation of the projectile and the gas volume $V(t)$ is then given by

$$V(t) \overset{\text{mod}}{=} V(t_0) + A\,s(t), \dot{v}_p(t) \overset{\text{mod}}{=} (1/m_p)[A\,p_b(t) - f_f(s(t)) - A\,p_a], \dot{s}(t) = v_p(t),$$
$$\dot{v}_r(t) = -(1/m_r)(A\,p_r(t)), \dot{s}_r(t) = v_r(t) \tag{2.9}$$

where

$A = \pi(C/2)^2$: Inside cross section area of the gun barrel; C is the caliber
$V(t_0)$: The initial volume of the cartridge
$v_p(t)$: Velocity of the projectile in a fixed system, $v_r(t)$: Velocity of recoil system
$s(t)$: Position of the rear of the projectile along the gun barrel direction
$s_r(t)$: Position of the recoil system
m_p : Mass of the projectile, m_r : Mass of the recoil system
f_f : Friction force of the projectile

p_a : Pressure in front of the projectile during motion in the gun barrel

p_b : Pressure at the base of the projectile, p_r : pressure working on the recoil system

Typically $m_p = 4.3\,10^{-2}\,kg$, $V_0 = 1.67\,10^{-5}\,m^3$, which gives that $\rho_s(t_0) = 0.9\,10^3\,kg/m^3$.

The mathematical model is equipped with an equation of state of the gas (Nobel-Abel for example) and of the solid (Compaction) of the form

$$
\begin{aligned}
p_g &\overset{\text{mod}}{=} n\,R\,T_g/(V_g(1-\mathrm{co}r(\rho_g)) = n'\,R\,T_g\rho_g/(1-\mathrm{co}r(\rho_g)), R = 8.31J/mol/K \\
\mathrm{co}r(\rho_g) &\overset{\text{mod}}{=} 1-Exp[-\rho_g/\rho_{g\max}], \rho_{g\max} = 1100kg/m^3, n' = 40mol/kg \qquad (2.10) \\
p_s &\overset{\text{mod}}{=} f_s(\rho_s)
\end{aligned}
$$

where

$\mathrm{co}r(\rho_g) \overset{\text{def}}{=} \rho_g \,\mathrm{cov}(\rho_g)$, cov( ) is called the co-volume of the gas particles
n :Number of mole's
n' :Number of mole's pr mass unit
T_g :Temperature of the gas
R :Universal gas constant

Observe that for small densities, $\rho_g/\rho_{g\max} \ll 1$. Then the approximation $\mathrm{cov}(\rho_g) \approx 1/\rho_{g\max}, \mathrm{co}r(\rho_g) \approx \rho_g/\rho_{g\max}$ can be used. During normal gun barrel motion this is a good approximation since the maximum pressure is around 400MPa with a corresponding density of $0.4\,10^3\,kg/m^3$, which gives that $\rho_g/\rho_{g\max} = 0.36$, this value differs from the exact value by only about 5%.

For small densities the equation of state of the gas can be written as

$$
p_g/\rho_g = n'\,R\,T_g + p_g/\rho_{g\max} \qquad (2.11)
$$

By using closed bomb measurements the so-called impetus $n'\,R\,T_g$ and the so-called co-volume $1/\rho_{g\max}$ can be read off from the graph by the relation ship (2.11). Measurements reported in [1] give the values in equation (2.10) when the temperature is calculated by using thermodynamic models [1].

The gas pressure does not work on the projectile since some of the solid particles occupy a space inside the volume V. The following relation is proposed for the average pressure

$$\overline{p} \overset{\text{mod}}{=} p_g agr(\rho_s) + p_s(1 - agr(\rho_s)), \quad agr(\rho_s) = Exp[-\chi\rho_s / \rho], \chi \, \square \, 1 \tag{2.12}$$

Observe that $\overline{p} = p_g$ if $p_g = p_s$, which is a good approximation for most gunpowder reaction since only the gas compresses the particles. Generally, when the solid density is low the solid pressure contribution is low, and the gas pressure distribution is high. When the solid density is high the solid pressure distribution is high, and the gas pressure distribution is low. The parameter χ is of the order of one.

During motion of the projectile along the gun barrel a rarefraction wave appears in the gas. The wave starts at the rear of the projectile, effectively decreasing the pressure working on the projectile. Using a quasi-static model it follows that [2]

$$p_b \overset{\text{mod}}{=} \overline{p}[1 - (1/2)(\gamma_g - 1)v_p / c_g]^{(2\gamma_g/(\gamma_g - 1))} \tag{2.13}$$

where

$c_g = (\gamma_g \partial p_g / \partial \rho_g)^{1/2}$: The adiabatic sound speed of the gas

The IBHVG98 code assumes that[1]

$$p_b \overset{\text{mod}}{=} [\overline{p} + M_g(p_a + f_f / A)/(3\, m_p)]/[1 + M_g/(3\, m_p)] \tag{2.14}$$

A shock wave also moves in the air in front of the projectile with a velocity larger than the projectile. The pressure of a quasi-static situation is constant from the front of the projectile and up to the shock front moving ahead of the projectile. The standard quasi-static formula gives the pressure, density and the position of the shock as provided in reference [2]

$$\begin{aligned}
p_a &\overset{\text{mod}}{=} p_0[1 + \gamma_a(\gamma_a + 1)v_p^2/(4c_a^2) + \gamma_a\, v_p / c_a(1 + (\gamma_a + 1)^2 v_p^2 / 16 / c_a^2)^{1/2}] \\
\rho_a &= \rho_0(p_a / p_0)^{(1/\gamma)}, \\
v_{sh} &= [(1/4)(\gamma_a + 1)v_p + ((1/16)(\gamma_a + 1)^2 v_p^2 + c_a^2)^{1/2}], \dot{s}_{hs} = v_{sh}
\end{aligned} \tag{2.15}$$

where

p_0 : Pressure in the air
c_a : Adiabatic sound speed in air
$\gamma_a = 1.4$: Adiabatic constant of air

---

[1] We have not found the physical assumptions supporting this model

v_{sh} : Shock velocity
s_{sh} : Shock position

The temperature of the gas has to be found as a function of time. A change in the internal energy during a short time step Δt is given as

$$\Delta(c_{vg}M_g(t)T_g(t)) = c_{vg}T_g^0\Delta M_g(t) - p_a\ A\ v_p\Delta t - \dot{Q}_f(t)\Delta t - \dot{Q}_h(t)\Delta t - \dot{Q}_k(t)\Delta t - \dot{Q}_r\Delta t, \qquad (2.16)$$

where

$\dot{Q}_f$:Heat loss due to work against the friction forces applied on the projectile
$\dot{Q}_h$:Heat loss due to heat flux into the gun barrel
$\dot{Q}_k$:Heat loss due to increased kinetic energy of the gun barrel gasses and solid
$\dot{Q}_r$:Heat loss due to increased rotational energy of the projectile
$c_{vg} = 1450 J/kg/K$: Heat capacity of the gas pr unit mass
$T_g^0 = 2950K$:Initial gas temperature immediately after decomposition

Taking the limit as the time step approaches zero, gives that

$$c_{vg}M_g(t)\dot{T}_g(t) \overset{\text{mod}}{=} c_{vg}\dot{M}_g(t)(T_g^0 - T_g(t)) - A\ p_a\ v_p - \dot{Q}_f(t) - \dot{Q}_h(t) - \dot{Q}_k(t) - \dot{Q}_r(t), \qquad (2.17)$$

Let $f_f(s)$ be the friction force between the gun barrel (cartridge initially)) and the projectile as a function of the position of the rear of the projectile along the gun barrel. Usually the force starts with a non-zero value since the cartridge compresses the projectile. Thereafter a larger force appears when the projectile is pressed and deformed as it enters the gun tube. Thereafter a smaller friction force appears as the projectile moves along the gun tube. The energy flux and the force is given as

$$\dot{Q}_f \overset{\text{mod}}{=} f_f(s)\,v_p, s \le s_{\max} \qquad (2.18)$$

where

$s_{\max}$: The maximum displacement of the rear of the projectile along the tube.

The heat loss due to energy flux from the gun gases into the gun barrel is very important to model since approximately 20% of the total energy of the gun powder gases are fluxed into the gun tube. Let T_s be the temperature at one specific point along the gun barrel at the surface between the gases and the tube. During a shot the temperature first increases due to frictional heating caused by the projectile. Thereafter the temperature increases due to interaction with the gun powder gases. The complete solution demands that the energy flux within the tube during the motion of the projectile is calculated. Simulations show that the temperature does not change too much, and it is assumed that the temperature in the tube is approximately constant during a shot. The heat flux into the gun barrel is then modelled as

$$\dot{Q}_h \overset{\text{mod}}{=} \sigma\,\varepsilon(T_{aw}{}^4 - T_s{}^4)2\pi(C/2)s + h_g(T_{aw} - T_s)2\pi(C/2)s, \qquad (2.19)$$
$$\varepsilon = 0.5, \sigma = 5.6\,10^{-8}W/(m^2 s), T_s = constant$$

where

ε: The emissivity of the gunpowder gases
σ: Stefan-Boltzmanns constant
T_{aw}: The adiabatic wall temperature
h_g : The effective heat transfer coefficient
$C/2$: The inside radius of the gun barrel; half of the calibre

T_s: The temperature of the gun tube

The adiabatic wall temperature is higher than the temperature at the edge of the boundary layer. The temperature of the gasses inside the boundary layer increases due to compression of the gas from the outside of the boundary layer to a position inside the boundary layer. The magnitude is dependent of the relative velocity between the hot gases and the tube, and varies accordingly along the gun barrel. Assuming an average value gives the following formulae of a turbulent boundary layer

$$T_{aw} \overset{\text{mod}}{=} T_g[1 + Pr^{1/3}(1/2)(\gamma_g - 1)(v_p/2)^2/c_g^2], \qquad (2.20)$$
$$h_g \overset{\text{mod}}{=} 0.05(Pr)^{1/3}(\rho_g v_p/(2\mu_g))^{0.8}\kappa_g/s^{0.2}$$

where

μ_g: Viscosity of the gasses
κ_g: Conductivity of the gasses
Pr: Prandtl number
h_g: Heat transfer coefficient
T_{aw}: The adiabatic wall temperature

$\gamma_g \overset{\text{def}}{=} 1 + n'R/c_{vg} = 1.23$: The adiabatic constant

$c_g = (\gamma_g \partial p_g / \partial \rho_g)^{1/2}$: The adiabatic sound speed

The IBHVG98 code use a very old formula from the second world war[2] written as

$$h_g \overset{\text{mod}}{=} (13.2 + 4\, Log_{10}[100\, C])^{-2} \rho_g (v_p / 2)\, c_{vg}\, \gamma_g / (\gamma_g - 1) \tag{2.21}$$

Observe that the heat conductivity of the gas is absent in the formulae!

Using the Boltzmanns equation the following approximations can be used

$$\begin{aligned}
&\mu_g \overset{\text{mod}}{=} 0.3\, \pi^{-1/2} (\overline{m}_g\, k\, T_{aw})^{1/2} / (\overline{A}_g), \kappa_g \overset{\text{mod}}{=} (\mu_g / \overline{m}_g) \overline{c}_g (1 + (9/4) k / \overline{c}_g) \\
&Pr \overset{\text{def}}{=} (\mu_g / \kappa_g) \overline{c}_g / \overline{m}_g = 1/(1 + (9/4) k / \overline{c}_g), k = 1.38\, 10^{-23} J/K, N_a = 6.02\, 10^{23} / mol \\
&\overline{m}_g \overset{\text{def}}{=} 1/(n' N_a) = 4.15\ \ 10^{-26} kg, \overline{c}_g \overset{\text{def}}{=} c_{vg} \overline{m}_g = 6.02\ \ 10^{-23} J/K, \\
&\overline{A}_g \overset{\text{mod}}{=} 4\pi [3/(4\pi) \overline{m}_g / \rho_{g\max}]^{2/3} = 4.3\ \ 10^{-20} m^2
\end{aligned} \tag{2.22}$$

where

$\overline{m}_g$:Average mass of molecular particles in the gas
$\overline{A}_g$:Average scattering surface of the molecular particles in the gas
$\overline{c}_g$:Average heat capacity pr molecular particles
k : Boltzmanns constant

N_a :Avogadros number

Observe that using the proposed approach, kinetic parameters as the viscosity and the heat conduction are connected to the thermodynamic properties. Of special importance is the relation for the average surface scattering area.

Assuming that the velocity of the gases and solid at each instant increases linearly with the distance from the rear of the gun barrel up to the projectile, gives that

$$m_{gs}(x) \overset{\text{mod}}{=} (M/s)x, v_{gs}(x) \overset{\text{mod}}{=} (v_p/s)x, x \le s \tag{2.23}$$

where

$v_{gs}(x)$: Velocity of the gas and solid inside the gun barrel as a function of the distance

[2] This seems to be an empirical formulae

$m_{gs}(x)$: Mass of the gas and solid inside the gun barrel as a function of the distance

Then the momentum m_k and the energy of the gas and solid follows by using (2.23) as

$$m_k = \int_0^s m_{gs}'(x) v_{gs}(x) dx = \int_0^s (M/s)(v_p/s) x dx = (1/2) M\ v_p,$$
$$Q_k = (1/2)\int_0^s m_{gs}'(x) v_{gs}(x)^2 dx = (1/2)\int_0^s (M/s)(v_p/s)^2 x^2 dx = (1/6) M\ v_p^2 \tag{2.24}$$

The time derivative gives

$$\dot{m}_k(t) = (1/2) M \dot{v}_p(t), \dot{M} = 0$$
$$\dot{Q}_k = (1/3) M v_p(t) \dot{v}_p(t) \tag{2.25}$$

The rotational energy of the projectile is given as

$$Q_r = (1/2) I \omega^2, \omega = 2\pi n / s_{\max} \dot{s}, \Rightarrow \dot{Q}_r = (2\pi n / s_{\max})^2 I\ \dot{s}\ \ddot{s} \tag{2.26}$$

where

$2\pi n / s_{\max}$: The number of radians pr length

I: Moments of inertia

Typically $I = m_p (C/2)^2$.

The pressure working on the recoil system can now be found. Using that the total momentum of the projectile, the gas, the gun and the air is conserved gives the total momentum of the gun powder gas, the projectile and the air as

$$m_t(t) = m_p(t) v_p(t) + (1/2)\ M\ v_p(t) + \rho_a(t)(s_{sh}(t) - s(t)) A\ v_p(t) \tag{2.27}$$

where

$m_t(t)$: total momentum of the projectile and the gases

The solid particles of the gun powder are assumed to follow the gas stream. Since momentum is conserved the recoil system must pick up this momentum. Assuming that the mass of the recoil system is m_r gives that the velocity of the recoil system is given by

$$m_r v_r(t) = m_r v_r(t_0) - (1/2)M\ v_p(t) - m_p v_p(t) - A\rho_a(t)(s_{sh}(t) - s(t))v_p(t) \tag{2.28}$$

where $v_r(t_0)$ is the initial velocity of the recoil system (in the direction of the velocity of the projectile).

Taking the time derivative gives that the acceleration of the recoil system is given by

$$\begin{aligned} m_r\dot{v}_r &= -m_p\dot{v}_p(t) - (1/2)\,M\ \dot{v}_p(t) - A\,\partial/\partial t\,[\rho_a(t)(s_{sh}(t) - s(t))v_p(t)] \\ &= -(1 + M/(2m_p))[A\ p_b(t) - f_f(s(t)) - A\ p_a] - A\,\partial/\partial t\,[\rho_a(t)(s_{sh}(t) - s(t))v_p(t)], \end{aligned} \tag{2.29}$$

which means that the “pressure “working on the recoil system is

$$\begin{aligned} p_r &= (1 + M/(2m_p))[p_b(t) - f_f(s(t))/A - p_a] + \partial/\partial t\,[\rho_a(t)(s_{sh}(t) - s(t))v_p(t)] + p_a \\ &= p_b(t) + M/(2m_p)[p_b(t) - f_f(s(t))/A - p_a] - f_f(s(t))/A + \partial/\partial t\,[\rho_a(t)(s_{sh}(t) - s(t))v_p(t)] \end{aligned} \tag{2.30}$$

The pressure working on the cartridge is called the breech pressure and is given by

$$p_{br} = p_r + f_f(s(t))/A \tag{2.31}$$

Assuming that the momentum of the recoil system is damped by an elastic device with stiffness K gives that

$$(1/2)m_r v_r(t_{\max})^2 = (1/2)K\ x^2, F = K\ x \Rightarrow F \propto (m_r v_r(t_{\max})^2)^{1/2} \tag{2.32}$$

where F is the force necessary to stop the motion of the recoil system. Observe that the force is proportional with the maximum velocity of the recoil system and also proportional with the square root of the mass of the recoil system.

3 SIMULATIONS

Figure 3.1 shows the pressure as a function of position. Figure 3.2 shows the velocity as a function of time. Figure 3.3 shows the temperature of the gas. Observe that the pressure has a maximum after a displacement of 5 cm in the gun barrel. The particles are burnt out at 0.5 m. The temperature as a function of position increases very fast (over a few mm.), thereafter it decreases.

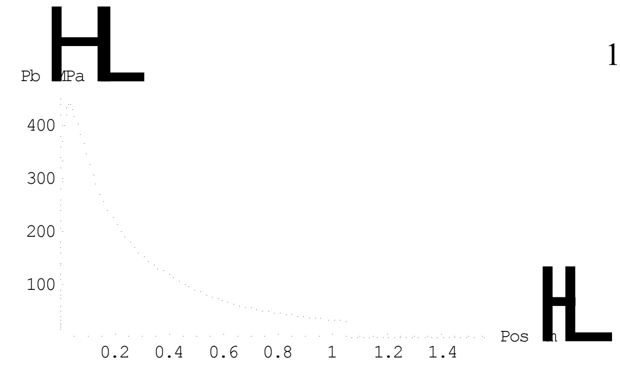

Figure 3.1 The pressure at the rear of the projectile as a function of position

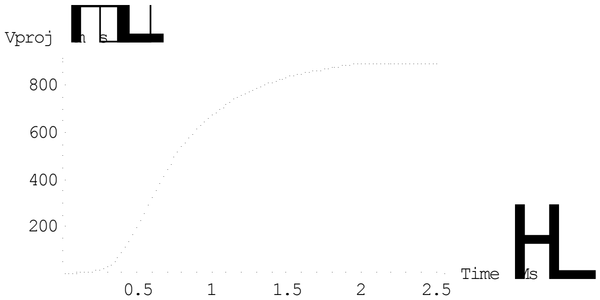

Figure 3.2 The velocity of the projectile as a function of time

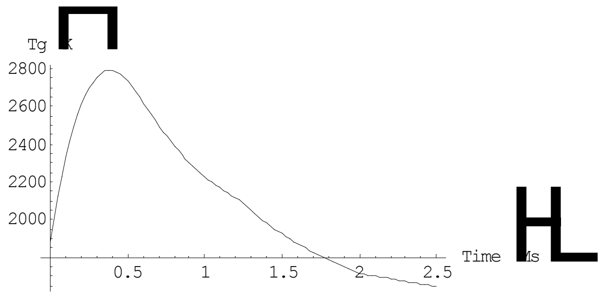

Figure 3.3 The temperature of the gas as a function time

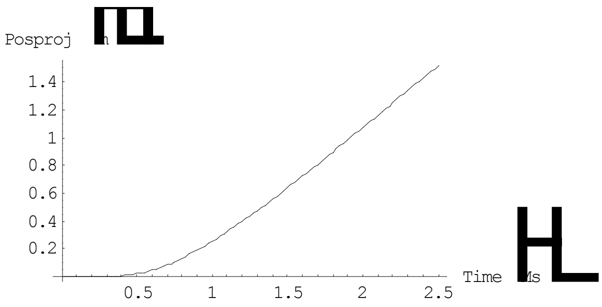

Figure 3.4 The position of the projectile as a function of time

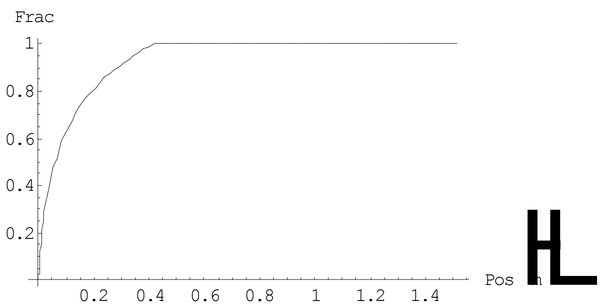

Figure 3.5 The fraction of unburned particles as a function of position

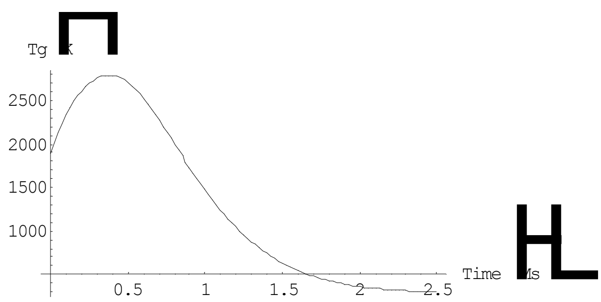

Figure 3.6 The temperature of the gas as a function time

Instead of using the formulas (2.14) and (2.21) as assumed in the IBHVG code, formulas, (2.13) and (2.20) are used. The pressure time history and the temperature time history are different, while the velocity history is more equal. 900m/s is also the experimental results. Figure (3.6) shows the temperature. Observe the difference between figure (3.3) and figure (3.6).

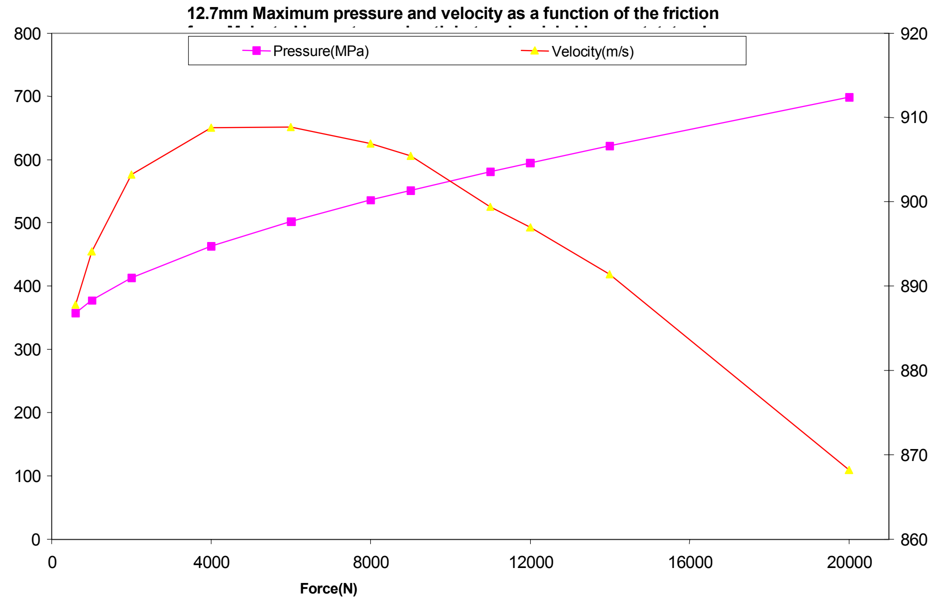

Figure 3.7 Effects from engraving and sedimentation in the gun barrel [1]. Left axis is Breech pressure in MPA. Right axis is exit velocity in m/s.

Experimental measurements of the gas pressure in guns show some scatter in the order of 50MPa at specific temperatures. The force necessary force the projectile along the gun barrel can change due to engraving of steel or due to sedimentation of brass in the gun barrel. Also the force necessary to drag the projectile out of the cartridge can vary between 1000 - 4000 N. Figure (3.7) shows curves from [1] where the friction force varies.

4 CONCLUSION/DISCUSSION

In this article a study of the pressure generated by burning of gun gases in gun barrels has been carried out. Calculations using the implemented burn model were compared with experimental results.

The numerical calculations gave good agreement with experiments of the gas pressure in a gun barrel, showing that the implemented slow burn model is viable and can be used in parametric studies. The observed scattering of some experimental results are discussed. Also some critical physical assumptions in the IBHVG98 code are pinpointed.

References

[1] G.O. Nevstad, Interiol Ballistics Properties of 12.7 mm MP ammunition, FFI/Rapport-2001/05000.

[2] L.D. Landau and E.M. Lifshitz, Course of Theoretical Physics, Fluid Mechanics, Pergamon Press 1982.

A APPENDIX

In order to completely present the time evolvement, a stepwise algorithm is given below. Let's say that all quantities are given at time t. In order to find the quantities at time $t+\Delta t$ the following algorithm is proposed:

First the new positions and velocities are calculated by

$$\begin{aligned}
&v_p(t+\Delta t) = v_p(t) + (\Delta t / m_p)(A\ p_b(t) - f_f(s(t)) - A\ p_a(t)), s(t+\Delta t) = s(t) + \Delta t\ v_p(t),\\
&v_r(t+\Delta t) = v_r(t) - (\Delta t / m_r) p_r(t), s_r(t+\Delta t) = s_r(t) + \Delta t\ v_r(t),\\
&v_{sh}(t+\Delta t) = [(1/4)(\gamma_a + 1)v_p + ((1/16)(\gamma_a + 1)^2 v_p^2(t+\Delta t) + c_a^2)^{1/2}], s_{sh}(t+\Delta t) = s_{sh}(t) + \Delta t\ v_{sh}(t),\\
&V(t+\Delta t) = V(t_0) + A\,[s(t+\Delta t) - sh(t+\Delta t)]
\end{aligned} \tag{A1}$$

The new burning fronts are calculated as

$$\begin{aligned}
&b(t+\Delta t) = b(t) + \Delta t\ H(\beta p_g(t)),\\
&l(t+\Delta t) = Max[0, L - 2b(t+\Delta t)], r(t+\Delta t) = Max[0, R_2 - R_1 - 2b(t+\Delta t)]
\end{aligned} \tag{A2}$$

The burn fraction equation gives the new burn fraction at time $t+\Delta t$ as

$$F(t+\Delta t) = 1 - r(t+\Delta t) l(t+\Delta t) / [(R_2 - R_1)L] \tag{A3}$$

The masses of gas and solid are then calculated as

$$M_s(t+\Delta t) = M_s(t_0)(1 - F(t+\Delta t)), M_g(t+\Delta t) = M - M_s(t+\Delta t) \tag{A4}$$

The new densities are given as

$$\begin{aligned}
&\rho_s(t+\Delta t) = M_s(t+\Delta t) / V(t+\Delta t), \rho_{pa}(\overline{p}(t)) = constant\\
&\rho_g(t+\Delta t) = M_g(t+\Delta t) / [V(t+\Delta t) - M_s(t+\Delta t) / \rho_{pa}(\overline{p}(t))]
\end{aligned} \tag{A5}$$

Thereafter, the new gas temperature is calculated as[3]

$$\begin{aligned}
&T_g(t+\Delta t) = T_g(t) + [M_g(t) T_g(t) + (M_g(t+\Delta t) - M_g(t)) T_g^0] / M_g(t+\Delta t) -\\
&(\Delta t / (c_{vg} M_g(t))(A\ p_a(t) v_p(t) + \dot{Q}_f(t) + \dot{Q}_h(t) + \dot{Q}_k(t) + \dot{Q}_r(t))
\end{aligned} \tag{A6}$$

---

[3] This routine gives a robust algorithm for the first part.

Thereafter the new gas and solid pressure, and adiabatic sound speed are calculated as

$$
\begin{aligned}
& p_g(t+\Delta t) = n' R\, T_g(t+\Delta t)\rho_g(t+\Delta t)/(1-\mathrm{co}\,r(\rho_g(t+\Delta t)) \\
& \mathrm{cov}(\rho_g(t+\Delta t)) = 1 - Exp[-\rho_g(t+\Delta t)/\rho_{g\max}],\, p_s(t+\Delta t) = f_s(\rho_s(t+\Delta t), \\
& c_g^2(t+\Delta t) = \gamma_g n' R\, T_g[1+\rho_g(t+\Delta t)/\rho_{g\max}]/(1-\mathrm{co}\,r(\rho_g(t+\Delta t))
\end{aligned}
\tag{A7}
$$

Thereafter the average pressure is calculated as

$$
\begin{aligned}
& \overline{p}(t+\Delta t) = p_g(t+\Delta t)agr(\rho_s(t+\Delta t)) + p_s(t+\Delta t)(1-agr(\rho_s(t+\Delta t))), \\
& agr(\rho_s(t+\Delta t)) = Exp[-\chi\rho_s(t+\Delta t)/\rho(t+\Delta t)]
\end{aligned}
\tag{A8}
$$

This article assumes that $\overline{p}(t+\Delta t) \approx p_g(t+\Delta t)$. The new pressures working at the base and in the front of the projectile is given by

$$
\begin{aligned}
& p_a(t+\Delta t) = p_0(1+\gamma_a(\gamma_a+1)v_p^2(t+\Delta t)/(4c_a^2) + \gamma_a\; v_p(t+\Delta t)/c_a(1+(\gamma_a+1)^2 v_p^2(t+\Delta t)/16/c_a^2)^{1/2}], \\
& \rho_a(t+\Delta t) = \rho_0(p_a(t+\Delta t)/p_0)^{(1/\gamma_a)}, \\
& p_b(t+\Delta t) = \overline{p}[1-(1/2)(\gamma_g-1)v_p(t+\Delta t)/c_g(t+\Delta t)]^{(2\gamma_g/(\gamma_g-1))}
\end{aligned}
\tag{A9}
$$

The recoil "pressure" is given as

$$
\begin{aligned}
& p_r(t+\Delta t) = p_b(t+\Delta t) \\
& +M_g(t+\Delta t)/(2m_p)[p_b(t+\Delta t) - f_f(s(t+\Delta t))/A - p_a(t+\Delta t)] - f_f(s(t+\Delta t))/A \\
& +(1/2)(M_g(t+\Delta t) - \dot{M}_g(t))v_p(t+\Delta t)/\Delta t/A \\
& \quad +\{[\rho_a(t+\Delta t)(s_{sh}(t+\Delta t) - s(t+\Delta t))v_p(t+\Delta t)] - [\rho_a(t)(s_{sh}(t) - s(t))v_p(t)]\}/\Delta t
\end{aligned}
\tag{A10}
$$

The heat loss due to friction is given as

$$
\dot{Q}_f(t+\Delta t) = f_f(s(t+\Delta))v_p(t+\Delta t),
\tag{A11}
$$

The new loss due to kinetic energy of the gun gasses and solid is

$$
\dot{Q}_k(t+\Delta t) = (1/3)Mv_p(t+\Delta t)\,(v_p(t+\Delta t) - v_p(t))/\Delta t
\tag{A12}
$$

The new adiabatic wall temperature is given as

$$
T_{aw}(t+\Delta t) = T_g(t+\Delta t)[1+Pr^{1/3}(1/2)(\gamma_g-1)(v_p(t+\Delta t)/2)^2/c_g^2(t+\Delta t)]
\tag{A13}
$$

The new conductivity and viscosity are calculated as

$$\mu_g(t+\Delta t) = 0.3\,\pi^{-1/2}(\overline{m}_g\ k\,T_{aw}(t+\Delta t))^{1/2}/(\overline{A}_g),$$
$$\kappa_g = (\mu_g(t+\Delta t)/\overline{m}_g)\overline{c}_g(1+(9/4)k/\overline{c}_g) \quad \text{(A14)}$$

The new heat transfer coefficient is given as

$$h_g(t+\Delta t) = 0.05(Pr)^{1/3}(\rho_g(t+\Delta t)v_p(t+\Delta t)/(2\mu_g(t+\Delta t))^{0.8}\kappa_g(t+\Delta t)/s(t+\Delta t)^{0.2} \quad \text{(A15)}$$

The new heat loss due to transfer into the tube is

$$\dot{Q}_h(t+\Delta t) = \sigma\ \varepsilon(T_{aw}(t+\Delta t)^4 - T_s{}^4)2\pi(C/2)s(t+\Delta t)$$
$$+h_g(t+\Delta t)(T_{aw}(t+\Delta t)-T_s)2\pi(C/2)s(t+\Delta t), T_s = constant \quad \text{(A16)}$$

The loss due to rotational energy is given as

$$\dot{Q}_r(t+\Delta t) = (2\pi n/s_{\max})^2 I\ v_p(t+\Delta t)\,(v_p(t+\Delta t)-v_p(t))/\Delta t \quad \text{(A17)}$$

The complete cycle is now finished!

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